MATHEMATICS CURRICULUM GUIDE

CONTENTS

MATHEMATICS CURRICULUM GUIDE FOR
GRADES 1 AND 2

Standard 3: Mathematics
Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability and trigonometry.

INTRODUCTION

Problem solving should be integrated throughout the standards. The development of problem solving skills should be a major goal of the mathematics program at every grade level. Students should engage in many problem-solving situations and have the opportunity to reflect upon their solutions. They should have opportunities to work collaboratively with other students as well as work individually. Students at this level should have opportunities to apply mathematics to real-life situations and express their understanding in a variety of modes (drawings, words, symbols and with concrete objects). They should engage in hands-on conceptual learning and have opportunities to use calculators and computers. However, the facility in the use of technology should not be a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

The curriculum for Grades 1 and 2 focuses on whole numbers includes counting, basic facts, addition and subtraction to 3-digit numbers, an introduction to multiplication and division, properties of addition and multiplication and estimation techniques. The study of rational numbers includes an introduction to fractions, ratio and probability. Students should engage in using mathematical language and symbols when sorting objects, comparing sets of objects, recognizing and describing simple patterns. The focus in geometry should be activities that develop spatial sense, recognizing two and three-dimensional shapes and an introduction to line symmetry. Students should use both standard and nonstandard units of measure limiting standard measurement to the metric system. Students should have opportunities to collect a variety of data, record it and create and do simple interpretations of pictographs and bargraphs. Students should start making predictions about and conducting simple probability experiments.

Manipulatives are a crucial element of instruction at this level because they allow students to form basic understandings vital to concept development. It is not necessary to devote large amounts of money on the purchase of commercial manipulatives. There are a variety of objects like beans and pasta that can be used as counters (an important manipulative at this level). Although classrooms need measuring devices like meter sticks, rulers can be duplicated on paper copiers and simple weighing devices can be made out of clothes hangers and margarine tubs. Pattern blocks, tangrams, pentominoes, square tiles and fraction models can be made from construction paper and laminated for long term use. Base 10 place value models at this level should be proportional (e.g. the object for the tens place should be ten times larger than the object for the ones place). Students can use bundles of coffee stirrers or base 10 blocks that are duplicated on a paper copier. Versatile commercial materials include connecting cubes, geoboards, meter sticks, thermometers, measuring cups and spoons, dice and calculators. Three-dimensional solids like prisms, pyramids, cylinders, etc. are also useful. Calculators need only be 4-function at these grade levels.

Assessment needs to be an ongoing process and not an end in itself. Teachers should monitor not only student progress but also the effectiveness of instruction. There are many methods to access student achievement. Paper and pencil tests, although useful, should not be the sole evaluation instrument. Teachers should use interviews, portfolios, class discussions, as well as observations to determine student understanding. Class discussions should not be confined to students answering recall questions but should evolve around open-ended questions that get students to explain their conceptual ideas.

The performance indicators, skill/concepts, sample tasks and assessment items that follow are suggestions and meant to be a guide for use by local education agencies in developing their own curriculum. The order and placement of topics does not imply the way it must be done for classroom instruction. They are not meant to be restrictive. We have used the following sources for ideas for sample tasks and assessment items. Teachers may wish to refer to them for more ideas.

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Baratta-Lorton (1976). *Mathematics Their Way.* Menlo Park, CA: Addison-Wesley Publishing Company.

Burns, Marilyn (1992). *About Teaching Mathematics: A K-8 Resource.* White Plains, NY: Math Solutions Publications.

Kennedy, Leonard, & Steve Tipps (1997). *Guiding Children's Learning of Mathematics, 8th Edition.* Albany, NY: Wadsworth Publishing Company.

Ohanian, Susan (1992). *Garbage Pizza Patchwork Quilts and Math Magic.* New York, NY:W.H. Freeman and Company

National Council of Teachers of Mathematics (1991) *Addenda Series, Kindergarten Book.* Reston, VA:NCTM

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-*Addenda Series, Second Grade Book.* Reston, VA: NCTM

New York State Education Department. (1992). *Mathematics K-6: A Recommended Program for Elementary Schools.* Albany, NY:NYSED

-(1989). *Suggestions for Teaching Mathematics Using Laboratory Approaches, Grades 1-6: 1. Number & Numeration.* Albany,NY: NYSED

-(1984). *Suggestions for Teaching Mathematics Using Laboratory Approaches Grades 1-6: 3. Geometry.* Albany, NY:NYSED.

-*Manipulative Materials Make Math More Meaningful.* Albany, NY: NYSED

(1996). *Leaning Standards for Mathematics, Science, and Technology.* Albany, NY: NYSED

Schielack, Jane & Dinah Chancellor (1995). *Uncovering Mathematics With Manipulatives and Calculators. Level 1.* Texas Instruments.

Whitin, David & Sandra Wilde (1992). *Read Any Good Math Lately?* Portmouth, NH: Heinemann.

Whitin, David & Sandra Wilds (1995). *It's the Story That Counts.* Portsmouth, NH: Heinemann.

3.1 Mathematical Reasoning 1-2
Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

ASSESSMENT TASK [3.1.4]
(2 points)
There is a ride at the amusement park that has cars and trucks that go around in a circle. There are 12 vehicles on the ride and there are 4 more cars than trucks. Use your manipulatives to find how many of each kind there are and then draw a picture to show your answer.

Performance Indicator	**Concepts/Skills**	**Sample Tasks**
3.1.1. Use models, facts and relationships to draw conclusions about mathematics and explain their reasoning	*1.* Compare sets of objects using terms: *more than, bigger than, greater than, less than, one more than, the same size, equal to, before, after* and *between.* 2. Demonstrate how addition and subtraction are opposite operations. *3.* Relate multiplication to repeated addition and counting by 2's, 3's, 4's etc. Multiplication and division are opposites. Relate division to repeated subtraction. *4.* Explore division as a process of sharing. *5.* Categorize objects using attributes such as likenesses and differences in color, shape, and size. *6.* Observe likenesses and differences using two categories at a time.	Using most calculators students can skip count by 1's, 2's, 3's, 4's, etc. for example by first entering + 2 and then an = to perform each repeated addition. The students can be asked to keep track of the repeated addition with counters and record how many times they added two for each new sum. The calculator can also perform repeated subtractions. For example, to demonstrate 12 divided by 4 as repeated subtraction, the child would first enter 12, then - 4 = and the = for each repeated subtraction until the display reads 0. The child keeps track of how many times 4 is subtracted before 0 is reached. [3.1.1.3]

3.1.2. Use patterns and relationships to analyze mathematical situations	*1.* Patterns for sums and differences using concrete materials, tables, calculators and number lines. *2.* Number sequences and patterns in the range 1-1000. *3.* Patterns of numbers that add up to a specific sum (e.g. all combinations of numbers that add to 6). *4.* Use patterns and relationships to discover commutative and associative properties, identity elements and to justify solution processes. *5.* Integrate the language of fractions with the language of probability. *6.* Use metric measures and money problems to reinforce place value	Provide students with a specified amount of two-color counters in a bag. Students shake and then spill the counters, keeping track of different combinations of red and yellow faces up. For example, a child may be given 6 counters. After the counters are spilled the counters may look as below- ●●●●○○ 4 + 2 = 6 Children would record the different combinations they find and stop when they believe they found all of them. Explore with the entire class whether 4 + 2 and 2 + 4 are the same combination. [3.1.2.3.]
3.1.3. Justify their answers and solution processes	1. Justify the reasonableness of answers. 2. Most topics can be used in problem solving	Give students cut-outs of ladybugs and a workmat with a large leaf and a small leaf. Ask each group to estimate the number of ladybugs needed to cover the smaller leaf. Have each group talk about their answer and agree on an estimate, then cover the smaller leaf with ladybugs. Have students guess how many ladybugs will be needed to cover the large leaf then cover it with their ladybugs. Have them discuss whether their second estimate for the was closer than their estimate for the smaller leaf and why. [3.1.3.1; 3.4.3.2; 3.5.4.1; 3.6.1.2; 3.6.4; 3.6.5.1]

3.1.4. Use logical reasoning to reach simple conclusions	1. Use concrete objects, pictorial representations, tables and number lines to represent and solve problems	Have students use counters to solve the following problem. A pet storeowner sold only birds and cats. One day he asked his clerk to count how many animals there was in the store. The clerk told him he counted 18 legs. • What is the least number of cats that could be in the store? • What is the least number of birds that could be in the store? • How many cats and birds might there have been? • Could there be more than one combination of animals in the store? [3.1.4.1; 3.3.1.2; 3.3.4.1]

3.2 Number and Numeration 1-2

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

ASSESSMENT ITEM [3.2.2; 3.2.3]
(3 POINTS)
Make as many three-digit numbers as possible from the digits 1,5, and 9, then put them in order from smallest to largest and explain in words, with pictures or with numbers how the numbers are the same and different from each other.

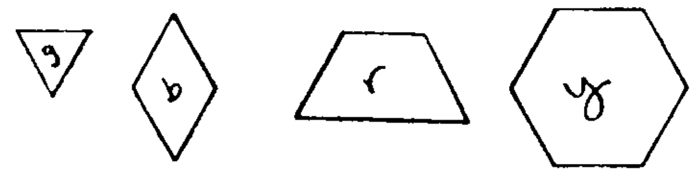
Numbers in order from smallest to largest.

How the numbers are the same?

How the numbers are different from each other?

Performance Indicator	Concepts/Skills	Sample Tasks
3.2.1. Use whole numbers and fractions to identify locations, quantify groups of objects and measure distances.	1. Arrangement of elements does not change the cardinal number-i.e. elements can be matched in a one-to-one correspondence. 2. Identify number names orally through 100 3. Ordinal numbers *first* to *thirty-first* 4. Count occurrences with tallies	Provide groups of students with about 20 buttons with one that is much larger or more ornate than the others. The special button represents the King who will eventually stand alone. The first student starts at the king and begins counting to 10 touching each button clockwise while counting. The student picks up the tenth button. The next student starts counting where the first student stopped and counts to ten and picks up the tenth button. If the tenth button is the king, the student cannot pick up the button and is out of the game. Keep playing until only one student is left. That student is the winner. [3.2.1.1; 3.2.1.2]
3.2.2. Use concrete materials to model numbers and number relationships for whole numbers and common fractions, including decimal fractions	1. Count forward and backward by 1's, 2's, 3's, 4's, 5's, 10's in various ways 2. Represent 2 and 3 digit numbers up to 999 using concrete models including the place of zero in the place value system 3. Concept of even and odd 4. Halves (thirds, etc.) of a whole are equal to each other 5. Unit fractions 1/2, 1/3, 1/4, 1/5, 1/6, 1/8, 1/10, 1/100 as part of a whole or part of a collection of things. 6. A set of objects can be divided into	Have students use counters to represent both even and odd numbers. They should try to put them into two equivalent rows. For example- ●●● ●●●● ●●● ●●● 6 7 Students discuss the differences between even and odd numbers [3.2.2.3]

	many equal parts.	
3.2.3. Relate counting to grouping and to place-value	1. Rename numbers 2. Meaning of digits in 3-digit numbers. 3. Expanded notation for 2 and 3-digit numbers	Provide students with unifix cubes and place value charts. Using a spinner with numbered sectors 1-9 or 0-9, the teacher spins a number and students represent the number with unifix cubes, write it on their place value chart and write the numeral. As teacher spins, students count on more unifix cubes and are directed to always make a rod of 10 when they can. On their place value mats they will write the new number and write the new numeral. Students could also be asked to show the new numbers in expanded notation such as 1 ten + 1 one. [3.2.3.2; 3.2.3.3]
3.2.4. Recognize the order of whole numbers and commonly used fractions and decimals	1. Count forward and backward 2. Concepts of *first, last,* and *middle* 3. Whole number immediately *before* or *after*. Whole number *between* two numbers	Have five students act out the following poem. Five little pumpkins are sitting on a gate. The first one said," Oh my, it's getting late" The second one said, "There are witches in the air." The third one said, "But we don't care." The fourth one said, "Let's run and run and run." The fifth one said, "I'm ready for some fun!" "Whoooo-oo-oo" went the wind and out went the lights. And the five little pumpkins rolled out of sight. • What happened first in the story? What happened last? • Who comes next after the second pumpkin? • The last pumpkin is in what place? • If Sally is the third pumpkin which one will come next? [3.2.4.2; 3.2.4.3; 3.2.1.5]

3.2.5. Demonstrate the concept of percent through problems related to actual situations.	1. Relate many-to-one in preparation for the concept of ratio 2. Concept of ratio	Using pattern blocks: green triangle, blue rhombus, red trapezoid and yellow hexagon, have students discover the amount of triangles needed to cover the blue rhombus, the red trapezoid and the yellow hexagon. Have them extend the ratios for multiple shapes.  [3.2.5.1; 3.2.5.2]

3.3 Operations 1-2

Students use mathematical operations and relationships among them to understand mathematics.

ASSESSMENT ITEM [3.3.1.2]
(2 POINTS)
Which is the greater sum or are the sums the same: 28 + 53 or 27 + 54?

Explain your answer with pictures, words or numbers.

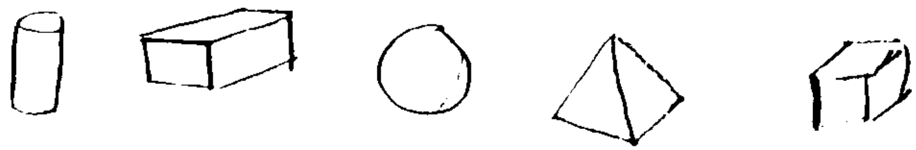
Performance Indicator	Concepts/Skills	Sample Task
3.3.1 Add, subtract, multiply, and divide whole numbers	1. Add and subtract 2 and 3-digit numbers with no regrouping 2. Add and subtract 2-digit numbers requiring regrouping	Provide students with coffee stirrers or craft sticks, rubber bands and a place value chart. Have them represent problems like 203 + 123 using single sticks for the units, a bundle of ten sticks for ten and a bundle of ten bundles of

	3. Combine sets to produce a new set	ten for hundreds. Students combine the coffee stirrers to find the sum and show the addition on the place value chart. [3.3.1.1; 3.3.1.3]
3.3.2 Develop strategies for selecting the appropriate computational and operational method in problem-solving situations .	1. Share sets of objects such as cookies, crayons. 2. Explore division as a process of sharing 3. Explore division as a process for finding the number of equivalent subsets in a given set	Use the story *A Doorbell Rang* by Pat Hutchins to help students understand division as a process of sharing. The book begins with two children who are about to share 12 cookies. Just as they are about to share the cookies, the doorbell rings and two friends join them. Now there are 4 people to share 12 cookies. Then two more friends arrive and now there are 6 children to share the cookies with. The doorbell rings again and 6 more children are there. Now there are 12 children sharing 12 cookies. Have students use their counters to demonstrate each situation and decide how many cookies each child will get in each situation. [3.3.2.1; 3.3.2.2]
3.3.3 Know single digit addition, subtraction, multiplication, and division facts	1. Special role of zero 2. Master addition facts with sums 0-18 and subtraction with differences 0-18 3. Readiness activities with rectangular arrays of objects 4. Repeated addition or counting activities 5. Multiplication and division facts through 25	Give students specified numbers of color tiles for example 18 tiles. Have them make as many rectangles as possible out of the tiles and record each rectangle on a piece of graph paper noting the number of rows and columns of each rectangle to find all the multiplication facts for the given number. [3.3.3.3; 3.3.3.5]
3.3.4 Understand the commutative and associative properties	1. Associative property of addition 2. Commutative property of addition 3. Commutative property of	Students can demonstrate the commutative property of multiplication by using unifx cubes to make for example 2 rods of 3 and place it over 3 rods of 2 noting that they are

	multiplication	equivalent lengths. [3.3.4.3]

3.4 Modeling/Multiple Representation 1-2
Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

ASSESSMENT ITEM [3.4.5]
(2 POINTS)
Circle the shape below that you would use to package your product if you were a manufacturer of tennis balls?



Explain why you would use that shape.

Performance Indicator	**Concepts/Skills**	**Sample Tasks**
3.4.1 Use concrete materials to model spatial relationships	1. Make geometric pictures, patterns, and designs using geometric shapes 2. Make designs using congruent and non-congruent shapes	Read *Grandfather Tang's Story* by Ann Tombert showing the students the tangram pictures. Provide the children with tangrams so they can make the animals in the story. [3.4.1.1]
3.4.2 Construct tables, charts, and graphs to display and analyze real-world data	1. Record information with tallies, blocks and pictographs	Have students use unifix cubes to make a bar graph showing the colors of their shoes. They can answers questions like • How many colors of shoes are we wearing? • Which color shoes are the most popular? [3.4.2.1]

3.4.3 Use multiple representations (simulations, manipulative materials, pictures, and diagrams) as tools to explain the operation of everyday procedures	1. Compare dimensions of various objects using terms like *longer than, taller than, smaller than, shorter than, as long as, farther, nearer* 2. Concepts of *more, less, the same*	Students are given a number of two-color counters. They spill them and record which color comes up more or if there is an equal number of each color. They keep track by making tallies under categories of MORE RED, SAME, MORE YELLOW.[3.4.3.2]
3.4.4 Use variables such as height, weight, and hand size to predict changes over time	1. Compare heights and duration of time in general terms	Students plant bean seeds and measure their growth at the end of every week. [3.4.4.1]
3.4.5 Use physical materials, pictures, and diagrams to explain mathematical ideas and processes and to demonstrate geometric concepts	1. Observe objects in the environment that have geometric shapes 2. Identify shapes in everyday life: circles, squares, rectangles, triangles 3. Terms *inside, outside* 4. Discover properties of 3-D shapes 5. Compare attributes of objects-size, shape, weight, texture, etc. 6. Draw symmetrical designs 7. Examine bilateral symmetry by paper folding or mirror activities	Provide students with pattern blocks and mirrors. Have them use the mirror to identify which shaped tiles have symmetry (bilateral). Then have them make a design with their patterns blocks that will be symmetrical. [3.4.5.6; 3.4.5.7]

3.5 Measurement 1-2

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

ASSESSMENT ITEM [3.5.1; 3.5.3; 3.5.6]
(2 POINTS)
Use the chart to answer the questions below.

Object	Weight
Pencil	12 g
Nickel	5 g
Penny	4 g
Sue's Purse	800 g
Wendall's Shoes	410 g
Notebook	350 g

What items weigh less than 20 grams?____________________
What items weigh more than 25 grams but less than 100 grams?____________________
Name an item that would weigh about as much as Sue's purse. ____________________

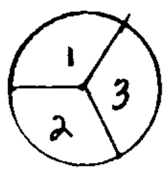
Performance Indicator	Concepts/ Skills	Sample Tasks
3.5.1. Understand that measurement is approximate, never exact	1. Compare (weather, food,…temperatures) in general terms 2. Develop an understanding of the need for standard units of measure	Give students a variety of objects in pairs and have them use balance scales to determine which of the two objects is heavier. [3.5.1.1]
3.5.2. Select appropriate standard and nonstandard measurement tools in measurement activities	1. Explore various nonstandard measurement tools such as blocks, books, children's feet, bowlful	Have students trace two copies of their foot and cut them out. Have them lay the "feet" end to end to measure the length of the room. Have them compare their measurements. Discuss why the measurements were not the same.[3.5.2.1]

3.5.3. Understand the attributes of area, length, capacity, weight, volume, time, temperature, and angle	1. Compare the capacity of containers using sand and water 2. Weighing experiences using the terms *heavier* or *lighter than* 3. Time to the hour, day, month and year using a clock and calendar 4. Time in half hours, quarter hours, minutes, and seconds 5. Weigh objects using grams 6. Make change for amounts of money up to $1.00, using pennies, nickels, dimes, quarters, and half dollars. 7. Introduce the kilogram, liter and Celsius thermometer	Give students a variety of different sized and shaped jars. Ask them to put them in order for the one that holds the least to the one that will hold the most. After the students have put their jars in order, have them test their order by pouring rice (sand or water) from the smallest jar to the next sized jar. If the rice spills out of the second jar then the order was not correct at this point. Have them continue the process through their entire series of jars. [3.5.3. 1]
3.5.4. Estimate and find measures such as length, perimeter, area, and volume using both standard and nonstandard units	1. Measure objects using nonstandard units 2. Estimate sizes using phrases like *about as long as, almost as long as, wider than*... 3. Use meter, centimeter and decimeter for measuring length 4. Recognize English system of measure only as it appears incidentally	Let students make copies of them selves using adding machine tape. They can work in pairs to tear off lengths of tape equal to their height and then tear off other pieces for their arms, legs and other body parts which they may wish to include. The teacher can ask them to compare the different parts of their body-Is your arm about as long as your leg? Etc. Using a mirror students can put the body parts together to make a copy of themselves. [3.5.4.1; 3.5.4.2]
3.5.5. Collect and display data	1. Collect objects of all sorts 2. Collect data concerning body measurements and other things of interest to the students 3. Simple bar graphs using stacks of blocks	Every month the children can make a class graph for which they must make a decision such as "Would you rather make a jack-o'-lantern with a happy face or a sad face? They could use self drawings on the graph each month and place their picture over their preferance. The picture graphs can be reinterpreted as bar graphs using

		connecting cubes that can then be counted by grouping cubes into groups of 10 to reinforce place value concepts. [3.5.5.2; 3.5.5.3]
3.5.6. Use statistical methods such as graphs, tables and charts to interpret data	1. Compare data in terms of number, equality, inequality, similarities, differences	Let students bring their favorite book to class and show them to each other. Have them discuss ways in which the books are similar or different. Have them choose one category of difference and sort the books on those categories. The books can be put on a large floor grid and the teacher can then ask questions about the number of books in each category.

3.6 Uncertainty 1-2
Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

ASSESSMENT ITEM [3.6.6; 3.6.8]
(3 POINTS)
If three children are playing a game in which Mike wins if the spinner lands on 1, Kathy wins if it lands on 2 and Irene wins if it lands on 3, would the game be fair and why?
(insert spinner with equal sized sectors)



Would the game be fair? Yes No
Why or why not?

What if Irene decided she did not want to play anymore and left and Mike decided that he would win if the spinner lands on 1 and 3 and Kathy wins if the spinner lands on 2. Is the game fair now? Why or why not?

Performance Indicator	Concepts/Skills	Sample Tasks
3.6.1 Make estimates to	1. Estimate quantities	Show students a jar filled with linking cubes. Have them

compare to actual results of both formal and informal measurements	2. Make estimating an integral part of measurement activities	estimate how many cubes are in the jar. Have them put their estimates on post-it notes and create a class graph of their estimates. When counting out the number of connecting cubes in the jar, link them in rods of ten to reinforce place value concepts. [3.2.3.2; 3.2.3.2; 3.5.5.3; 3.6.1.1]
3.6.2 Make estimates to compare to actual results of computations.	1. Estimate the answers before solving problems	Introduce front end rounding for addition and multiplication. For example in the example 25 + 32, the student estimates that 20 + 30 is 50 so the answer must be greater than 50. [3.6.2.1]
3.6.3 Recognize situations where only an estimate is required	1. Investigate various numerical problems that arise in school.	Have students discuss questions such as- • Would it be all right to estimate the number of our milk order today? Why or why not? • Would it be all right to estimate how many students are going on the class field trip? Why or why not? • Would it be okay to estimate how many cupcakes to bring to a party? [3.1.1.1; 3.1.1.4; 3.6.3.1]
3.6.4. Develop a wide variety of estimations skills and strategies	1. Use manipulative materials for estimating quantity.	When estimating how many candies in a jar, let students count how many are on the bottom of the jar and then estimate how many layers there are. They can perform repeated addition to find an estimate of the number of candies in the jar. [3.6.4.1; 3.6.1.1]
3.6.5 Determine the reasonableness of results	1. Anticipate outcomes by guessing or estimating	Have students guess how many cut out lady bugs they could put on a small leaf and then check by putting as many as possible on the leaf. Give them larger ladybugs and ask them if they would be able to use the same

		number, less or more of the larger ladybugs to completely cover the leaf and explain their reasoning. [3.1.3.1; 3.4.3.2; 3.5.4.1; 3.6.1.2; 3.6.4.1; 3.6.5.1]
3.6.6 Predict experimental probabilities	1. Discuss certainty or uncertainty of events 2. Terms *more likely* or *less likely* 3. Discuss fairness of a game 4. Predict outcomes of coin tosses	Let students examine paper cups to decide how they might land when dropped either on their sides, on their tops or on their bottoms. Have them predict which outcome is most likely and test it. [3.6.5.1; 3.6.6.2; 3.6.8.1]
3.6.7 Make predictions using unbiased random samples	1. Make predictions	Give students bags with different colored jellybeans in them. Have them reach in the bag, pull out a jellybean, record its color and then replace it. After about 10 samples have the student decide which color there is the most of in the bag. [3.6.7.1; 3.6.5.1]
3.6.8. Determine probabilities of simple events	1. Perform experiments with three or more likely outcomes 2. Use language such as "one chance in three" 3. Solve problems such as how many different pairs of numbers have a sum of 6 4. Concept of *combination* or *arrangement* 5. Solve problems such as "How many different sets of 3 numbers will add up to 12?" or "How many different ways can you rearrange the letters of your name?"	Ask students to find all combinations of outfits that could be made of 3 colors of shirts and 2 colors of pants. Have them use cut outs to make the outfits and different colored connecting cubes to record each combinations. Ask them to show how many different outfits they made and from that to determine how many chances out of the 6 would they pick one particular outfit. see the color. [3.6.7.1; 3.6.8.1; 3.6.8.2; 3.6.8.5]

3.7 Patterns/Functions K-2
Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

ASSESSMENT ITEM [3.4.1; 3.7.5]
(2 POINTS)
Draw the next 5 shapes in the pattern below.

▼▼◆▼▼◆▼▼◆

Performance Indicator	**Concepts/Skills**	**Sample Tasks**
3.7.1. Recognize, describe, extend, and create a wide variety of patterns	1. See 3.1.2 and 3.4.1	Using a Hundred Board and colored transparent chips, give the students the first three numbers of a pattern e.g. 1,3,5 or 6, 16, 26. Have them put a chip on each of the three numbers, showing the first three numbers of the pattern. Then ask the students to continue the pattern by showing the next three numbers. Help the students verbalize a rule for each pattern. [3.1.2.2; 3.2.2.1; 3.2.2.3; 3.2.4.1; 3.3.3.4; 3.7.1]
3.7.2 Represent and describe mathematical relationships	1. Symbols <, >, = 2. Read and write numerals 0-100 3. also 3.1.2, 3.2.2 and 3.4.3	Use tape to make a large square on the classroom floor. Divide the square into four equal parts and label each with a number from 1 to 4. Have children stand behind a throw line 6 feet from the target and take turns tossing a beanbag. The child who tosses must read the number and then perform some designated action such as stamping, clapping, hopping or jumping the number of times. [3.2.1.2; 3.7.2.2]
3.7.3 Explore and express relationships using variables and open sentences	1. Concepts of equality and inequality using number 2. Also 3.3.1 and 3.3.4	Using a mathematical balance, assign two children to work together. Place a weight on any number, on one side of the balance. How many ways can the scale

		balance if you use one weight? two weights? Three weights? Children should write the equations they discover on the balance. [3.3.1.3; 3.3.3.2; 3.7.3.1]
3.7.4 Solve for an unknown using manipulative materials	1. See 3.3.1, 3.3.2 and 3.3.4	Using a mathematical balance, discuss with the children the analogy of the balance fulcrum with the equal symbol in an equation. Have children correlate their equations with the balance. Give them open sentences such as 5 + ❑ = 9. Have them represent it with connecting cubes to find the missing addend and check their answer on the mathematical balance by placing a weight on the 4 on the same side as the 5 and seeing if it balances. [3.1.1.2; 3.1.3.1; 3.1.4.1; 3.3.3.2; 3.4.3.2; 3.7.4.1]
3.7.5 Use a variety of manipulative materials and technologies to explore patterns	1. See 3.1.2	Provide students with a blank 100s grid, counters and a calculator. Have them determine how many counters fit across the grid. At the same time have them count by one's on the calculator. Ask them what patterns they notice on the calculator as they fill up the grid. Have them hold as many beans as they can in their hand, then count how many they could hold. Have them use the calculator and the grid to determine how many they could hold in two hands. 3.1.2.1; 3.1.2.2; 3.2.2.1; 3.2.3.2; 3.2.4.1; 3.3.1.2; 3.5.2.1; 3.7.5.1]
3.7.6 Interpret graphs	1. See 3.4.2 and 3.5.6	Have students write their first name on a post-it note and count the number of letters in their name. Have the students come up to the board where the teacher has written different numbers (starting with 2) and have the students place their post-it note over the number which is the same as the number of letters in their name. Ask

		questions about who has the greatest number of letters, who has the least, who is in the middle, what is the most common number of letters in a name, etc. [3.4.3.1; 3.4.3.2; 3.5.5.2; 3.5.5.3; 3.5.6.13.7.6.1]
3.7.7 Explore and develop relationships among two- and three-dimensional geometric shapes	1. See 3.4.5	Provide each group of students with some geometric solids. Ask them to identify and share with the rest of the class the solids that answer each of these questions. • Think of a square. Find a block with a square face. (cube and rectangular prism with square bases) • Think of a rectangle. Find a block with a rectangular face (any prism) • How many different kinds of rectangular faces can you find? • Think of a triangle. Find all the blocks you can with triangular faces. (triangular prism, pyramid, thetrahedron) • How many triangular faces does each one have? [3.4.5.4; 3.7.7.1]
3.7.8 Discover patterns in nature, art, music, and literature	1. See 3.4.5	Give students an opportunity to look a number of quilt patterns. Possibly reading the book *The Keeping Quilt* by Patricia Polacco as an introduction. Let them choose one quilt to study and write about the pattern they notice. Using construction paper to make geometric shapes, have them create a block of their own quilt pattern. [3.4.1.1; 3.4.1.2; 3.4.5.1; 3.4.5.2, 3.87.8.1]

MATHEMATICS CURRICULUM GUIDE FOR
GRADES 3-4

Standard 3: Mathematics
Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability and trigonometry.

INTRODUCTION

Problem solving should be integrated throughout the standards. The development of problem solving skills should be a major goal of the mathematics program at every grade level. Students should engage in many problem-solving situations and have the opportunity to reflect upon their solutions. They should have opportunities to work collaboratively with other students as well as work individually. Students at this level should have opportunities to apply mathematics to real-life situations and express their understanding in a variety of modes (drawings, words, symbols and with concrete objects). They should engage in hands-on conceptual learning and have opportunities to use calculators and computers. However, the use of technology should not be a substitute for a student's understanding of quantitative concepts and relationships or proficiency in basic computation.

The curriculum focus for grades 3-4 on whole numbers includes: basic facts, addition, subtraction, multiplication, special role of zero and one in multiplication, division, properties of addition and multiplication and estimation techniques. The study of rational numbers includes fractions, ratio and probability. The focus on fractions includes ordering fractions, adding and subtracting fractions with unlike denominators and computation with decimal fractions. Students should engage in using mathematical language and symbols when sorting objects, comparing sets of objects, recognizing and describing simple patterns. The focus in geometry should be activities that develop spatial sense, recognizing two and three-dimensional shapes and an introduction to line symmetry. Students should use both standard and nonstandard units of measure limiting standard measurement to the metric system. Students should have opportunities to collect a variety of data, record it and create and do simple interpretations of pictographs and bargraphs. Students should start making predictions about and conducting simple probability experiments.

Manipulatives are a crucial element of instruction at this level because they allow students to form basic understandings vital to concept development. It is not necessary to devote large amounts of money on the purchase of commercial manipulatives. There are a variety of objects like beans and pasta that can be used as counters (an important manipulative at this level). Although classrooms need measuring devices like meter sticks and balance beams, rulers can be duplicated on paper copiers. Pattern blocks, tangrams, pentominoes, square tiles and fraction models can be made from construction paper and laminated for long term use. Base 10 place value models at this level should be proportional (e.g. the object for the tens place should be ten times larger than the object for the ones place). Students can use base 10 blocks that are duplicated on a paper copier. They can begin to use more abstract base 10 models that are not proportional. Versatile commercial materials include connecting cubes, geoboards, meter sticks, thermometers, measuring cups and spoons, dice and calculators. Three-dimensional solids like prisms, pyramids, cylinders, etc. are also useful. Calculators need only be 4-function at these grade levels.

Assessment needs to be an ongoing process and not an end in itself. Teachers should monitor not only student progress but also the effectiveness of instruction. There are many methods to access student achievement. Paper and pencil tests, although useful, should not be the sole evaluation instrument. Teachers should use interviews, portfolios, class discussions, as well as observations to determine student understanding.

Class discussions should not be confined to students answering recall questions but should evolve around open-ended questions that get students to explain their conceptual ideas.

The performance indicators, skill/concepts, sample tasks and assessment items that follow are suggestions and meant to be a guide for use by local education agencies in developing their own curriculum. The order and placement of topics does not imply the way it must be done for classroom instruction. They are not meant to be restrictive. We have used the following sources for ideas for sample tasks and assessment items. Teachers may wish to refer to them for more ideas.

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Van De Walle, John. (1990). *Elementary School Mathematics: Teaching Developmentally.* White Plains, NY: Longman.

		make if she uses all the shirts and shorts? Explain how you got your answer with a picture or a diagram. [3.1.3.1, 3.1.3.2, 3.1.4.1]	
3.1.4 Use logical reasoning to reach simple conclusions.	1. Use concrete objects, diagrams, open sentences, patterns, relationships and estimation to solve problem. 2. Identify the missing information needed to find a solution to a given story problem	Before a basketball game the five starting players shake the hand of every starting player on the other team. How many handshakes occur between the players? Have children estimate the number of handshakes, then use diagrams to answer. [3.1.3.1, 3.1.4.1, 3.1.4.2]	

3.2 Number and Numeration 3-4

Students use number sense and numeration to develop an understanding of thew multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS	CONCEPTS/SKILLS	SAMPLE TASKS	ASSESSMENT ITEMS
3.2.1 Use whole numbers and fractions to identify locations, quantify groups of objects, and measure distances.	1. Read and write whole numbers to hundred millions. 2. Cardinal numbers through 10,000 and ordinal numbers through 500. 3. Relate fractions to the monetary system and to metric measure. 4. Identify the use of fractions in daily life: a half-hour TV program, an eighth note, a quarter pound of butter, 1500 meter run	Give each group a newspaper, glue, crayons, and a copy of Newspaper Scavenger Hunt. Remind them to work together. NEWSPAPER SCAVENGER HUNT In a newspaper, find the following items, cut them out, and paste your examples next to the description. 1. The price of something to eat	Mr. Murray's class has been collecting marbles in a jar. They are going to have a pizza party when
3.2.2 Use concrete materials to model numbers and number relationships for whole numbers and common fractions, including	1. Use manipulative aids such as base 10 blocks or abaci to reinforce the concept of regrouping. 2. Consider, discuss and predict whether the sum, difference, or product of two numbers is odd or even.	2. A street address 3. A number that gives a size 4. A phone number 5. The date the paper was published 6. A number in a recipe 7. A number that names a distance	they get 1,000 marbles The chart shows how many marbles they have

decimal fractions.	3. Examine prime numbers less than 100 4. Skip count to numbers greater than 100. 5. Use metric measures to reinforce the understanding of place value. 6. Introduce places to the right of the decimal using money. 7. Show inverse relationships between multiplication and division involving fractions 8. Examine various ways that a figure can be divided into equal parts. Use the terms numerator and denominator. 9. Using manipulative materials to correlate common fraction notation for tenths, hundredths, and thousandths with decimal notation. 10. Order unit fractions using < and > signs utilizing concrete materials. One whole equals two halves, three thirds, etc. 11. Order common fractions and decimal fractions using < and > signs using concrete materials. Also recognize equivalent fractions. 12. Extend number line to include negative numbers.	8. A number that names a temperature 9. A number written in words 10. The score of a game To extend the activity have the students: - classify the numbers they found into two or more groups by circling the numbers with different colored crayons. - order the numbers from smallest to largest - write each number in 2 ways - write a paragraph on how numbers are used in their own lives Then have students explain and discuss the findings of the different groups. [3.2.1.1, 3.2.1.4, 3.2.2.6, 3.2.2.9, 3.2.2.11]	collected so far. MONTH MARBLES Sept. 127 Oct. 214 Nov. 275 Dec. 198 How many more marbles do they need in order to earn their pizza party?
3.2.3 Relate counting to grouping and place value.	1. Place value concepts to millions and hundredths.	Solve each of these problems and explain what happens to the number 37 in each of the situations. When is it appropriate to use decimals, fractions, or whole numbers in your answer? (1) Four children shared 37 dollars as equally as possible. (2) Four children shared 37 pennies as equally as possible.	

3.2.4 Recognize order of whole numbers and commonly used fractions and decimals.	1. Reinforce the understanding of numbers to 10,000. 2. Fractions should use common denominators of 2, 3, 4, 5, 6, 8, 10, 12.	(3) Four children shared 37 equal-sized squares of paper as equally as possible. (4) Four children in a relay team race want to beat the fastest score of 37 seconds. If each child runs the same amount of time, how many seconds will each person need to run the relay? [3.2.1, 3.2.3.1, 3.2.4]	
3.2.5 Demonstrate the concept of percent through problems related to actual situations.	1. Percents should be common and limited to multiples of 10 such as 10%, 50%, and 100% 2. Develop the concept of ratio in practical problem-solving situations.	Have students work in pairs with a flat and 100 small cuisenaire cubes. Tell them that the flat is a unit, or whole. Have them put 50 cubes side by side on top of the flat. Ask what part of the flat is covered by cubes. Write the common fraction and decimal fraction on the board. Write 50% next to the ½ and .50. Continue with other problems. [3.2.5.1]	

3.3 Operations 3-4

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS	CONCEPTS/SKILLS	SAMPLE TASKS	ASSESSMENT ITEMS
3.3.1 Add, subtract, multiply, and divide whole numbers.	1. Add and subtract whole numbers less than one million. 2. Regroup in subtraction when zeroes are in the minuend. 3. Multiplication and division facts through 144. 4. Multiply 3-digit numbers by 2-digit numbers. Multiplication by multiples of 10. 5. Short and long algorithms for dividing by a 1-digit divisor.	Using the digits 3, 4, and 5, place each digit in a box in order to get the largest possible answer (product). No digit may be used more than once. [3.2.3, 3.3.1.3, 3.3.1.4]	I have 11 coins worth $.62. Show one collection of 11

3.3.2 Develop strategies for selecting the appropriate computational and operational method in problem solving situations.	1. Use diagrams and open sentences to help solve word problems. 2. Use knowledge of operation concepts.	Shanelle earns $3.50 per hour for babysitting. Each week she babysits for 4 hours. A. How much money does she earn in 1 week? B. How much money does she earn in 4 weeks? [3.1.4.2, 3.3.2.2]	coins that is worth exactly $.62. [3.1.1.1, 3.1.1.4, 3.3.3.1]
3.3.3 Know single digit addition, subtraction, multiplication, and division facts.	1. Check computations by using inverse relationships of operations. 2. Special role of 0 and 1 in multiplication. 3. Recognize the number 1 as the identity element not only in multiplication, but also in division.	Your classmate thinks that any number times zero is just that number with a zero on the end, as in 5 x 0 = 50. Show and explain to your classmate what happens when any number is multiplied by zero. Use at least two examples. [3.3.3.2]	
3.3.4 Understand the commutative and associative properties.	1. Study the commutative property in sentences and multiplication tables. 2. Study the associative property of multiplication.	Two students copied three numbers from the chalkboard. Laurie wrote 18+7+24. Julie wrote 7+24+18. How many different ways are there of writing the problem? What is the sum in each case? [3.3.4.2]	

3.4 Modeling/Multiple Representation 3-4

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS	CONCEPTS/SKILLS	SAMPLE TASKS	ASSESSMENT ITEMS
3.4.1 Use concrete materials to model spatial relationships.	1. Investigate properties of plane figures. 2. Study properties of solid figures (vertices, line segments, edges, and angles). 3. Use geometric figures to make designs and patterns.	Show the class two solids, for example, a square prism and a triangular prism. Ask "How are these two solids the same?" "How are these two solids different?" Repeat these questions for two other solids. Give each child a set of solids (cube, triangular prism, rectangular prism,	Mr. Chin wants to build a

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MATHEMATICS CURRICULUM GUIDE FOR GRADES 7-8

Standard 3: Mathematics

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability and trigonometry.

INTRODUCTION

Problem solving should be integrated throughout the standards. The development of problem solving skills should be a major goal of the mathematics program at every grade level. Students should engage in many problem-solving situations and have the opportunity to reflect upon their solutions. They should be actively involved individually and in groups in exploring, conjecturing, analyzing, and applying mathematics in both a mathematical and a real-world context. Students should have opportunities to express their understanding in a variety of modes (diagrams, graphs, words, symbols, numbers and manipulatives) and to engage in hands-on conceptual learning. Technology used appropriately should assist the student in exploration activities and be used as a problem-solving device, not a substitute for the student's understanding of quantitative concepts and relationships or proficiency in basic computation.

The curriculum focus for grades 7 and 8 begins to emphasize real number skills as an integral part of problem-solving activities. Students should reason in spatial contexts, with proportions, from graphs, inductively and deductively. They should continue to develop number and operation sense through the creation of algorithms and procedures, the use of estimation in problem solving and checking the reasonableness of results, and exploring relationships among representations of, and operations on, integers and rational numbers. They should be able to identify and use functional relationships, develop and use tables, graphs, and rules to describe situations, and interpret among different mathematical expressions. When given a problem, students should be able to write an equation and find its solution. They should be able to use a variety of methods to solve linear equations and informally investigate inequalities and nonlinear equations. Students use statistics to describe, analyze, evaluate, and make decisions about data and extend the measures of central tendency to include grouped data using frequency tables. Students create experimental and theoretical models of situations involving probabilities. They should develop an understanding of geometric objects and relationships and use geometry, measurement and estimation to solve problems.

Manipulatives are a crucial element of instruction at this level especially when new concepts are being introduced. It is not necessary to devote large amounts of money on the purchase of commercial manipulatives. There are a variety of objects like rulers, meter sticks, tape measures, protractors, and graph paper that are inexpensive and easily obtainable. Spinners can be made with a paper clip and a pencil point. Compasses can be made with string and a pencil. Fraction tiles, square counters, pattern tiles, tangrams, and pentominoes can be made from construction paper. Base 10 place value models no longer need to be proportional but can include more abstract models as in chip trading activities. Versatile commercial materials include cubes, geoboards, dice, capacity measuring devices, and mirrors. Calculators should be able to perform fraction operations, powers and give very large or very small decimal numbers in scientific notation..

Assessment needs to be an ongoing process and not an end in itself. Teachers should monitor not only student progress but also the effectiveness of instruction. There are many methods to access student achievement. Paper and pencil tests, although useful, should not be the sole evaluation instrument. Teachers should use interviews, portfolios, class discussions, as well as observations to determine student understanding.

Class discussions should not be confined to students answering straight recall questions but should evolve around open-ended questions that get students to explain their ideas.

The performance indicators, skill/concepts, sample tasks and assessment items that follow are suggestions and meant to be a guide for use by local education agencies in developing their own curriculum. The order and placement of topics does not imply an expectation of how classroom instruction should be sequenced. They are not meant to be restrictive. We have used the following sources for ideas for sample tasks and assessment items. Teachers may wish to refer to them for more ideas.

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3.1 Mathematical Reasoning 7-8

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

PERFORMANCE INDICATORS	CONCEPTS/SKILLS	SAMPLE TASKS
3.1.1 Apply a variety of reasoning strategies.	1. Use pictures, diagrams, or patterns. 2. Use trial and error (guess). 3. Use a simpler but related problem. 4. Use proportional reasoning, ratios and rates to solve problems. 5. Work backwards.	"Force Out" is a game played with two people. Starting with any whole number, each player in turn must take away a single-digit number. The player who gets to 0 is the loser. (Someone should recognize that the strategy for winning is developed by thinking backward.) After figuring out the winning strategy, discuss who will win if both players know the winning strategy? Find the winning strategy if the players can only subtract even numbers. [3.1.1.5]
3.1.2 Make and evaluate conjectures and arguments using appropriate language.	1. Discriminate relevant from irrelevant information. 2. Alter the problem. 3. Seek a general solution. 4. Study cases where the general solution does not apply. 5. Explain and show solution processes in a variety of ways (words, numbers, symbols, pictures, charts, graphs, tables, diagrams and models. 6. Express solutions clearly and logically using appropriate mathematical notation, terms, and language.	Math Journal – Have students keep a math journal. The topics can be teacher generated or student free lance. Topics can be in a free writing style (without constraints) or formal (correct grammar and punctuation). Topics ideas: 1) Write 12 things you know about the number 1000. 2) Write a reaction to the lesson being studied. 3) "Today, I discovered …" 4) Why do you think integers were invented? [3.1.2.5]
3.1.3 Make conclusions based inductive reasoning.	1. Devise formulas (perimeter, area, circumference, etc) 2. Identify patterns in a number sequence (include sequences with integral terms). 3. Formulate properties (commutative, associative, etc) involving operations with integers by experimenting with integers under the basic operations. 4. Apply strategies and results from simpler problems to more complex situations 5. Discover rules of divisibility of numbers in the context of finding prime factors.	The table shows the height of a plant during a period of 3 weeks. Initially the plant was 5 inches tall. The chart indicates the growth of the plant for week 1 through week 3. Weeks (W) — Height (H) (in inches) 0 — 5 1 — 8 2 — 11 3 — 14 A. Write an equation that represents the height (H) of the plant in terms of the number of weeks (W). B. Use the table or your equation to predict the height of

		the plant after 10 weeks. [3.1, 3.4, 3.7]
3.1.4 Justify conclusions involving simple and compound (i.e., and/or) statements.	1. Find numbers that satisfy one or more conditions. 2. Support solutions with written and/or algebraic evidence.	Find the solution set for $n > -4$ and $n \leq 6$. Graph the solution set. [3.4.4, 3.7.5]

ASSESSMENT ITEM [3.1.3]

The variables a, b, and c each represent a different whole number. Use the properties of whole numbers to determine the values for each variable. Show you work.

$\frac{a}{a} = b$

$b + b = c$

$c \times c = a$

3.2 Number and Numeration 7-8

Students use number sense and numeration to develop an understanding of thew multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS	CONCEPTS/SKILLS	SAMPLE TASKS
3.2.1 Understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, expanded and scientific notation).	1. Read and write integers and rational numbers. 2. Approximate integers and rational numbers using scientific notation (positive and negative powers of 10) and explain the process. 3. Read and write irrational numbers 4. Understand the relationship between terminating and repeating decimals. 5. Describe the equivalent relationships among representations of rational numbers (fractions, decimals and percents) and use these representations in estimation, computation and applications. 6. Understand and explain a number raised to the zero power with use in exponents and exponential form. 7. Using real-life situations apply the concept of scientific notation to express and compare very large and very small numbers.	Working in small groups, have students find information (newspaper, reference material, textbooks) that can be written in scientific notation. They should find examples for which scientific notation can be used for numbers *larger than or equal to 1* and scientific notation for positive *numbers less than 1 but greater than zero.* Have them chart the information showing the data, actual number, and scientific notation. [3.2.1]

	8. Understand the meaning of the absolute value symbol.	
3.2.2 Understand and apply ratios, proportions, and percents through a wide variety of hands-on explorations.	1. Interprets percent as part of 100 using a variety of manipulatives (algebra tiles, graph paper, cubes). 2. Develop an understanding of the relationships between ratio, proportion and percent. 3. Calculate fraction, decimal, and percent equivalence. 4. Solve real life problems dealing with consumer affairs, scale drawings, similar polygons, distance and time. 5. Find the percent of a number; calculate the percent of increases and decreases, rate, commissions, taxes, and simple interest.	An inspector found 5 defective cassettes out of a random sample of 200 cassette tapes. If 4,000 cassette tapes are produced each day, how many tapes would you expect to be defective? A) Write a proportion that can be used to solve this problem. B) Solve the problem. [3.2.2.4, 3.3.1]
3.2.3 Develop an understanding of number theory (primes, factors, and multiples).	1. Define and identify prime and composite numbers. 2. Define and identify prime factors using factor trees and repeated division. 3. Factor numbers by using the rules of divisibility.	Have the class prepare a chart that shows all factors for each number 1 through 30 (number, factors, sum of factors) and post it in the room. Organize students into cooperative-learning groups. Give each group a paper that contains questions and have the recorder write the group's responses to each question. -Which numbers have two and only two factors? What kind of numbers are these? - Which numbers have three and only three factors? What will be the next number that has only three factors? - Which numbers have four and only four factors? The numbers 8 and 27 have exactly four factors. How do they differ from other numbers with four factors? Can you predict the next number of this kind? - Which numbers have five and only five factors? Can you describe these numbers? What is the next number that has exactly five factors? - Can you write a mathematical rule that tells how to determine the sum of factors when a number is prime? Can you write a rule for finding the sum when a number has exactly three factors? Discuss with the students the results of their work. [3.2.3]

3.2.4 Recognize order relations for decimals, integers, and rational numbers.	1. Compare and understand inter-relationships of integers and rational numbers. 2. Use symbols (<, >, = , ≤, ≥, ≅) when recognizing numerical relationships.	Give the student a list of real numbers and have them show their position on a number line. For example, show the position of the following on a number line: ¾, -1, √16, 2 1/8, √9, 2.6, 17/7 [3.2.4.2, 3.4]

ASSESSMENT ITEM [3.2.2, 3.4.2]

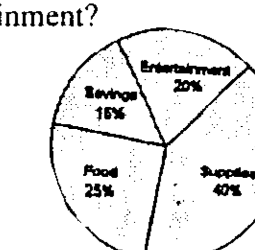
A scale drawing of a room uses a scale of 1 to 40. Which of the following proportions should be used to find the actual measures of the width, w, of the room which is 2.5 inches on the drawing?

A. $\frac{1}{40} = \frac{w}{2.5}$ C. $\frac{w}{2.5} = \frac{1}{40}$

B. $\frac{1}{40} = \frac{2.5}{w}$ D. $\frac{w}{40} = \frac{1}{2.5}$

3.3 OPERATIONS 7-8

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS	CONCEPTS/SKILLS	SAMPLE TASKS
3.3.1 Add, subtract, multiply, and divide fractions, decimals, and integers.	1. Consistently and accurately performs operations on integers, decimals, and rational numbers. 2. Raises rational numbers to whole number powers. 3. Determine the absolute value of real numbers expanded to include numerical expressions beyond a single value. 4. Solve one- and two-step word problems. 5. Solve one- and two-equations.	The graph below shows how Sue spent her allowance last week. If Sue's allowance is $6.00, how much of her allowance did she spend on entertainment? [3.3.3.1, 3.3.4, 3.3.5, 3.4.4] 

3.4 Modeling/Multiple Representation 7-8

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS	CONCEPTS/SKILLS	SAMPLE TASKS
3.4.1 Visualize, represent, and transform two-and three-dimensional shapes.	1. Identify and construct two-dimensional patterns for three-dimensional models 2. Recognize similarity and rotational and bilateral symmetry in two- and three-dimensional figures. 3. Identify elements of three-dimensional geometric objects.	Have students examine logos of businesses for rotational and bilateral symmetry. Then, using the computer, design a logo for an area business that is an example of rotational or bilateral symmetry. [3.4.1.2, 3.5.5, 3.7.6]
3.4.2 Use maps and scale drawings to represent real objects or places.	1. Students select appropriate units of measure and use proportional reasoning to convert measures. 2. Construct scale drawings and models with reasonable measurement accuracy.	Draw a map of the complete classroom to scale. Include windows, doors, desks and all objects in the room. Show the scale used to complete the exercise. [3.2.2, 3.3.1, 3.3.7, 3.4.2.2, 3.5.1, 3.5.2, 3.5.6]
3.4.3 Use the coordinate plane to explore geometric ideas.	1. Locate a point using ordered pairs of integers on the coordinate plane. 2. Locate the quadrant in which an ordered pair of integers is located. 3. Develop geometric ideas such as measurement formulas using geoboards and graph paper.	Have the student make a triangle within a rectangle on a geoboard and find the area of each figure by counting the number of squares contained in the figures. Then have the student develop the formulas for the area of each figure. [3.4.3.3, 3.5.3]
3.4.4 Represent numerical relationships in one- and two-dimensional graphs.	1. Use a number line graph to represent the solution of a problem with one unknown. 2. Use two-dimensional graphs, including the coordinate plane, to represent the solution of a problem.	Measure the heights of each student. Then have students graph the information using 3 different types of graphs. Discuss types of graph (stem and leaf, line, circle, etc) and the information they best portray. [3.4.4.2, 3.4.4.4, 3.5.4]

3.4.5 Use variables to represent relationships.	1. Use variables and appropriate operations to write an expression, equation, inequality, or system of equations or inequalities which represent a verbal description. (three less than a number, half as large as area A) 2. Interpret, demonstrate understanding and use of variables in expressions, formulas, equations, and properties.	Eye Spy : A student observes a tape measure which has been taped to a wall by looking through a tube (use different sizes: toilet paper, paper towel, gift wrap). The group identifies the lowest and highest measurements they are able to read from the tape. Another student records the distance between the two measures. The student moves increasing distances (100 cm, 150 cm, 200 cm and 300 cm) from the tape measure and repeats the observations. Each group should write a formula expressing the relationship between the distance, the tube, and the observations of the tape measure. A.) Solve, graph and explain in writing the outcome. B.) The class can discuss the different findings of each group and determine the differences and similarities of the expressions and findings. [3.4.5.1, 3.5.1, 3.5.2,3.5.3]
3.4.6 Use concrete materials and diagrams to describe the operation of real world processes and systems.	1. Model situations geometrically to interpret, formulate, and solve problems. 2. Compare geometric measurements and computations on coordinate axes as they are applied to parallel lines, congruent and similar figures.	Have students find the amount of wood needed to frame a 3' x 2.5' portrait, or find the number of plants that would fit in a six square foot garden plot if the plants were to be placed 6 inches apart. [3.3.1, 3.3.6, 3.4.6.1]
3.4.7 Develop and explore models that do and do not rely on chance.	1. Develop and explore combinations and permutations. 2. Construct an appropriate sample space (board games, spinners, dice, coins). 3. Explore the range of probabilities (certainty, impossibility, sometimes) 4. Consider the reliability of sampling procedures.	When people buy cars, they may study the reliability ratings of different models. Based on the personal experiences of car owners, cars are rated to be reliable or unreliable. Using *Consumer Reports* and other sources that provide background information on car safety have students decide what car they would like to own. Then have them research records to determine the reliability of the car they chose compared to two other models. If they were buying based on the data which model would they buy? Explain why? [3.1.2, 3.4.7.4]
3.4.8 Develop two- and three-dimensional transformations.	1. Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them 2. Identify similar and congruent shapes and determine their image under simple transformations (translation, rotation, reflection) in the plane.	Have students work in groups to design and complete a collage displaying objects (pictures, representations, or actual) that depict simple translations. [3.4.1,3.4.8.2, 3.7.8]

3.4.9 Use appropriate tools to construct and verify geometric relationships.	1. Identify and construct basic elements of geometric figures, (altitudes, midpoints, diagonals, angle bisectors and perpendicular bisectors; and central angles, radii, diameters and chords of circles). 2. Identify the properties of congruent and similar triangles. 3. Identify corresponding and vertical angles.	Three segments are given whose lengths are 2, 3, and 5 centimeters. Using any of the given lengths as many times as you wish: - How many equilateral triangles can be constructed? Construct one.
3.4.10 Develop procedures for basic geometric constructions.	1. Construct an angle with a given measure. 2. Bisect an angle using a compass and a straightedge. 3. Classify triangles according to angle size. 4. Classify triangles by the lengths of sides.	- How many isosceles triangles can be constructed? Construct one. - Is it possible to construct a triangle whose sides measure 2 cm, 3 cm, and 5 cm? Why or why not? [3.4.9, 3.4.10, 3.5.3, 3.7.7,]

ASSESSMENT ITEM [3.4.3]

On grid paper, plot the coordinates for the quadrilateral, connecting the points in order as you proceed. Be sure to connect the last point to the first point.

Quadrilateral: (-3,1), (-3, 6), (2, 6), (2, 1)

Name the type of quadrilateral that you drew and explain in words why it is this type of quadrilateral.

3.5 Measurement 7-8

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS	CONCEPTS/SKILLS	SAMPLE TASKS
3.5.1 Estimate, make, and use measurements in real-world situations.	1. Measure the distance of objects using scientific notation (shuttle from the earth). 2. Measure temperatures in different parts of the world and different planets. 3. Solve distance problems in miles per hour. 4. Use measurement in everyday situations.	Estimate and measure the surface areas of a set of gift boxes in order to determine how much wrapping paper will be required. [3.1.1,3.5.1.4]

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3.5.4 Use statistical methods and measures of central tendencies to display, describe, and compare data.	1. Interpret graphs, tables, scales, and charts by making identification comparisons, and calculations. 2. Use appropriate statistical measures to show data. 3. Determine which measures of central tendency (mean, median, mode) best represents the sets of data. 4. Organizes and displays collected data using appropriate tables, charts or graphs. 5. Apply data from measures of central tendencies to frequency tables. 6. Construct histograms and frequency polygons.	1. Choose a survey topic that applies to your class (height, time spent doing homework, watching TV). 2. Take a survey by collecting data from each member. 3. Organize the data, display the results on an appropriate graph. 4. Find the mean, median, and mode. 5. Write a report of your findings. [3.1.2, 3.3.1, 3.5.4.3]
3.5.5 Explore and produce graphic representations of data using calculators/ computers.	1. Use scientific and graphing calculators and computer spreadsheets to organize and analyze data.	Graph $y = x^2$ to show that a quadratic equation graphs a curve (parobala).
3.5.6 Develop critical judgement for the reasonableness of measurement.	1. Make an appropriate estimate relating to size, quantity, temperature, capacity and the passage of time. 2. Make measurement conversions between customary and metric units.	Estimate the volume of containers of different size and shape. Find the actual volume using measurement formulas. [3.5.6.1]

ASSESSMENT ITEM [3.5.4]

John recorded the following marks on his mathematics tests:

Test	1	2	3	4	5
Mark	95	80	85	95	90

What is John's median test mark?

A. 95 C. 89
B. 90 D. 80
E. None of these

3.5.4 Use statistical methods and measures of central tendencies to display, describe, and compare data.	1. Interpret graphs, tables, scales, and charts by making identification comparisons, and calculations. 2. Use *appropriate statistical measures to show data.* 3. Determine which measures of central tendency (mean, median, mode) best represents the sets of data. 4. Organizes and displays collected data using appropriate tables, charts or graphs. 5. Apply data from measures of central tendencies to frequency tables. 6. Construct histograms and frequency polygons.	1. Choose a survey topic that applies to your class (height, time spent doing homework, watching TV). 2. *Take a survey by collecting data from each member.* 3. Organize the data, display the results on an appropriate graph. 4. Find the mean, median, and mode. 5. Write a report of your findings. [3.1.2, 3.3.1, 3.5.4.3]
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B. 90
C. 89
D. 80
E. None of these

3.6.4 Use simulation techniques to estimate probabilities.	1. Conduct a variety of simulations to represent an experiment that can not be determined by empirical or theoretical probability.	Warren, Tom, Nancy, and Pat are infielders on a baseball team. There are two runners from the other team on second and third bases. With one out Happy Slugger comes up to bat. Given this information, make up a problem that can be solved by doing simulation. Then solve your problem by developing a simulation and carrying it out. Give your problem to others to solve. Compare strategies and answers. [3.6.4.1]
3.6.5 Determine probabilities of independent and mutually exclusive events.	1. Understand and use empirical and theoretical probability. 2. Predicts the results of a series of trials once the probability for one trial is known.	Students work in large groups with fair objects (coin, spinner, dice) to explore the probability that the use of cumulative relative frequency supports theoretical probability. First find the theoretical probability of the event. Then each person in the group attempts 20 trials. Organize the group's results in a table to find the cumulative relative frequency for each additional 20 trials. Did the cumulative relative frequency of the group support the theoretical probability finding. [3.6.5.1]

ASSESSMENT ITEM [3.6.3]

Joan has a choice of outfits consisting of one blouse, one skirt, and one scarf from those listed below:

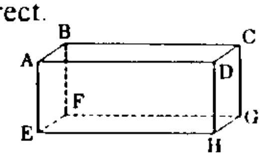
blouse	skirt	scarf
white	black	stripes
yellow	brown	plaid

How many different combinations of outfits consisting of one blouse, one skirt, and one scarf could Joan choose? What is the probability that the outfit chosen consisted of a plaid scarf, black shoes, and a blouse?

3.7 Patterns/Functions 7-8

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS	CONCEPTS/SKILLS	SAMPLE TASKS
3.7.1 Recognize, describe, and generalize a wide variety of patterns and functions.	1. Identify, describe, represent, extend and create patterns (numerical and geometric). 2. Develop the concept of function and the terminology by analyzing simple applications.	Have students work in groups to find the 100th term in the sequence of odd numbers starting from 1. Calculators may be used. [3.1.3, 3.2.3, 3.3.6, 3.7.11]
3.7.2 Describe and represent patterns and functional relationships using tables, charts, and graphs, algebraic expressions, rules and verbal descriptions.	1. Describe and represent numerical and geometric patterns and functions using equations, graphs and tables. 2. Organize and analyze data resulting in function applications through use of a table of values, sentence, formula, graph and prediction.	A student who does errands and yard work for a neighbor earns $3 per hour. Organize and analyze the data from the problem and prepare a formula describing the function in symbols) table of values showing how the two variables are related, and a sentence describing the pattern. Make a graph of the values that contains the appropriate labels and graph the solution. [3.1.2, 3.2.2, 3.3.7, 3.4.3]
3.7.3 Develop methods to solve basic linear and quadratic equations.	1. Use five basic properties of equality in solving equations with one variable. 2. Understand the addition, subtraction, multiplication and division properties as they pertain to problem-solving situations with inequalities. 3. Model and solve multi-step problems involving rate, average speed, distance and time, or direct variation. 4. Use algebraic expressions, equations and inequalities to model linear and nonlinear situations, including direct and inverse variation, exponential growth and quadratic behavior. 5. Fundamental ideas of the quadratic equation and its graph.	Students should know that linear situations "grow by adding," versus, for example, exponential situations, which "grow by multiplying," and recognize these characteristics in tables, graphs, equations and situations. Have students develop a problem that represents a linear or exponential situation or a functional relationship. Solve the problem. [3.2.2, 3.3.2, 3.3.6,3.7.3.4]
3.7.4 Develop an understanding of functions and functional relationships: that a change in one quantity (variable) results in change in another.	1. Examine a situation and extract from it quantities that vary directly and indirectly, and represent that variation graphically, in a table and in an equation. 2. Identify the input and the output in a relationship between two variables and determine whether the relationship is a function.	At a theater, the cost of utilities, film rental, and salaries is $180 per night. A ticket costs $4. Using x = number of ticket buyers and y = profit/loss, develop the function: $Y = 4x - 180$ A. Why is the graph acceptable in quadrants I and IV only? B. How many tickets must be purchased to meet expenses? To make a profit? [3.1.2, 3.4.3, 3.4.4, 3.7.4.1]
3.7.5 Verify results of	1. Solve the equation and check the solution set by	Solve linear equations, such as $2(x+3)=x+5$ by several

substituting variables.	substitution or graphing.	methods. Check by using substitution. [3.1.3, 3.1.4, 3.4.5, 3.7.5.1]
3.7.6 Apply the concept of similarity in relevant situations.	1. Demonstrate an understanding of congruence between two geometric figures and what congruence means about the relationships between the sides and angles of the two figures. 2. Understand the difference between similar and congruent.	Ask each student in the class to draw a triangle, using protractor and straight edge, so that two of its angles measure 90° and 30°. Compare various triangles drawn and use the definition to show why the triangles are similar. [3.7.6.2]
3.7.7 Use properties of polygons to classify them.	1. Identify similar and congruent triangles and other polygons and their corresponding parts.	Cut two similar triangles out of cardboard. Use these two models to show the figures formed by allowing the three pairs of corresponding angles to coincide. Select any model and practice writing proportions using the given lenghths. [3.7.7]
3.7.8 Explore relationships involving points, lines, angles, and planes.	1. Understand and use proper terminology and symbols, definitions, and formulas for undefined and defined terms 2. Name, define and measure angles and angle pairs.	Using the figure below have students answer questions and explain why their answer is correct.  a.) Points D, C, G, and ___ are coplanar. b.) B.) Which lines are parallel to EH? c.) Which plane is parallel to BCGF? [3.7.8]
3.7.9 Develop and apply the Pythagorean principal in the solution of problems.	1. Use the Pythagorean Theorem in the solution of problems (include rational and irrational numbers).	Mr. Modell drove his car 16 km north and then 12 km east. How far is he from his starting point? [3.5.3.2, 3.7.9, 3.7.10
3.7.10 Explore and develop basic concepts of right triangle trigonometry.	1. Understand the relationships of the sides of a right triangle. 2. Explore and develop that corresponding angles of similar triangle have the same measure. 3. Develop the formulas for sine, cosine and tangent.	To fly a kite, Stephen let out 240 meters of string. The string made an angle of 63^0 with the ground. How high, to the nearest meter, is the kite above the ground? [3.7.10.1]
3.7.11 Use patterns and functions to represent and solve problems.	1. Use patterns and functions to solve problems.	Working in pairs, students will make up two number Patterns following rules such as "3n - 1" and swap their patterns with another pair of students. Each pair must then identify the rule for each pattern the other pair created and justify their findings to the other pair. [3.4.5, 3.7.11.1]

ASSESSMENT ITEM [3.7.4]

Copy and complete the table below for the function:

$$y = 3x - 1$$

x	y
3	
	2
0	
-3	

On graph paper graph the function $y = 3x - 1$. Be sure to label 4 points with the coordinates from the table.

MATHEMATICS CURRICULUM GUIDE FOR MATH A

Standard 3: Mathematics

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability and trigonometry.

INTRODUCTION

Problem solving should be a means as well as a goal of instruction. Students should engage in many problem-solving situations and have the opportunity to reflect upon their solutions. They should be actively involved individually and in groups in constructing and applying mathematical ideas. Students should engage in a variety of instructional formats such as small groups, individual explorations, peer instruction, whole-class discussions and project work. They should be expected to communicate mathematical ideas in a variety of formats such as drawing diagrams, providing written explanations, using equations, inequalities, graphs, and manipulatives. They should recognize and apply the interrelatedness of mathematical topics and use technology as a tool for learning and doing mathematics. However, the use of technology should not be a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Teachers should use effective questioning techniques that promote student interaction and provide for the systematic maintenance of student learnings and embed review in the context of new topics and problem situations.

By the time students are ready to take the Math A exam they can determine, understand, apply and justify properties involving geometric figures. They pose, test and justify conjectures in algebraic, geometric, probabilistic and statistical contexts. Students write, simplify, evaluate and solve linear, quadratic, direct variation, exponential and other equations in applied and abstract contexts. Students can work with a variety of algebraic expressions, generalize exponent properties and use right-triangle trigonometry in applications.

Manipulatives continue to be a crucial element of instruction especially when new concepts are being introduced. Some useful manipulatives at this level include algebra tiles, miras, volume demonstration kits, probability tools like dice and spinners, metric measuring devices, two and three-dimensional geometric models, geoboards, tangrams, pentominoes, and tessellation tiles. Calculators should be a minimum of scientific but more useful would be graphing calculators as students need to understand the differences in a variety of types of functions.

The assessment of learning should be an integral part of instruction to inform the teacher of the effectiveness of instruction as well as student progress. Students should be expected to express their understanding with graphs, symbols and with written and oral descriptions. Class discussions should not be confined to recall questions but should evolve around open-ended questions that get students to explain their ideas.

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3.1 Mathematical Reasoning Math A
Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

ASSESSMENT ITEM [3.1.1; 3.1.2]
(4 points)
At Johnson High School, each student is required to belong to one of four clubs. The four clubs are Art, Chess, Debate, and Film. Bob, Jill, Rich, and Tony belong to four different clubs. Each of the statements below is true.

- Jill belongs to either the Film Club or the Debate Club
- Either Rich or Tony belongs to the Art Club.
- Jill does not belong to the Debate Club.
- Jill belongs to the Film club if and only if Tony belongs to the Chess club.

Use the information above to determine which club each student belongs to, and show how you arrived at your answer.

Performance Indicator	Concepts/Skills	Sample Tasks
3.1.1. Construct valid arguments	1. Truth value of compound sentences (conjunction, disjunction, conditional, related conditionals such as converse, inverse and contrapositive, and biconditional). 2. Truth value of simple sentences (closed sentences, open sentences with replacement set and solution set, negations).	Have students in a group design a survey containing five true-false questions and try out the survey on three people. Then have them redesign the survey replacing each statement with its negation and try the survey on five more people. Have students compare the results of the two surveys and decide why the results were different and then have each group of students report out to the rest of the class. [3.1.1.2]

3.1.2. Follow and judge the validity of arguments	1. Truth value of compound sentences (conjunction, disjunction, conditional, related conditionals such as converse, inverse and contrapositive, and biconditional).	Have students first fill in the following table and then discuss results in small groups. Suppose the employer tells Sam, "If you take this special course, then you'll get a raise of $100 a month." Assume Sam won't be angry unless the employer lied, and assume that the sentence is false if and only if the employer lied. <table><tr><td></td><td>Did the Employer Lie?</td><td>Is Sam Angry?</td><td>T or F</td></tr><tr><td>Sam takes the course. He gets the raise.</td><td>______</td><td>______</td><td>_____</td></tr><tr><td>Sam takes the course. He doesn't get the raise.</td><td>______</td><td>______</td><td>_____</td></tr><tr><td>Sam doesn't take the course. He gets a raise.</td><td>______</td><td>______</td><td>_____</td></tr><tr><td>Sam doesn't take the course. He doesn't get the raise.</td><td>______</td><td>______</td><td>_____</td></tr></table>[3.1.2.1]

3.2 Number and Numeration Math A

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

ASSESSMENT TASK [3.1.1; 3.2.1; 3.5.6]

(4 points)

A clothing store offers a 50% discount at the end of each week that an item remains unsold. Patrick wants to buy a shirt at the store and he says, "I've got a great idea! I'll wait two weeks, have 100% off and get it for free!" Explain to your friend Patrick why he is incorrect and find the correct percent of discount on the original price of a shirt.

Performance Indicator	Concepts/Skills	Sample Tasks
3.2.1. Understand and use rational and irrational numbers.	1. Real numbers 2. Irrational numbers including non-repeating decimals, irrational roots and pi.	Have students work in groups to solve the following problem, "How fast was that car going?" Suppose that the 4 tires of a car skid for L feet before stopping. Then let r be the speed of the car. Police officers sometimes estimate the speed a car was traveling by measuring the skid marks and using the following formulas. On a dry concrete road, $r \approx \sqrt{24L}$ On a wet concrete road, $r \approx \sqrt{12L}$ Do these formulas seem correct considering that a person would probably skid farther on wet pavement than on dry pavement? Justify your answer with graphs, words or numbers. [3.2.1.2; 3.6.1; 3.7.2.1; 3.7.3.1]
3.2.2. Recognize the order of real numbers	1. Rational approximations of irrational numbers	Draw a number line and indicate the position of the following real numbers on it using the letters as labels. A. 5 B. –8 C. 5/4 D. –0.75 E. $\sqrt{9}$ F. $-\sqrt{2}$ G. $\sqrt{3}$ H. π I. $\sqrt{49}/\sqrt{7}$ [3.2.2.1]
3.2.3. Apply the properties of real numbers to various subsets of numbers	1. Properties of real numbers including closure, commutative property, associative property, inverse element and distributive property.	Have students make multiplication and addition charts for a 12 hour clock, using only the numbers 1 –12. Have students determine if the system is closed under addition and multiplication. If not they should give a counter example. Determine if multiplication and addition are commutative under the system and if not give a counter example. Determine if there is an identity element for addition and multiplication and if so what are they? Determine if addition and multiplication are associative under the system and if not give a counter example. Does each element have an additive and multiplicative inverse?

		Determine if multiplication is distributive over addition (if not give a counter example) and if addition is distributive over multiplication (if not give a counter example. [3.2.3.1; 3.3.4]

3.3 Operations Math A

Students use mathematical operations and relationships among them to understand mathematics.

ASSESSMENT ITEM [3.3.1]
(4 points)
Express as a single fraction in lowest terms.

$$\frac{y-4}{2y} + \frac{3y-5}{5y}$$

Performance Indicator	Concepts/ Skills	Sample Tasks
3.3.1. Use addition, subtraction, multiplication, division and exponentiation with real numbers and algebraic expressions	1. Signed numbers 2. Use of variables: order of operations and evaluating algebraic expressions 3. Addition of polynomials: combining like terms and fractions with like denominators 4. Multiplication of polynomials: powers, products of monomials and binomials, equivalent fractions with unlike denominators, and multiplication of fractions 5. Simplification of algebraic expressions using addition and multiplication 6. Division of polynomials by monomials: powers, positive, zero and negative exponents, scientific notation, simplification of fractions, division of	Consider two consecutive positive integers. Which is larger, the average of their squares or the squares of their average? Show your reasoning algebraically. [3.3.1.4]

	fractions. 7. Prime factorization 8. Factoring common monomials 9. Binomial factors of trinomials 10. Binomial factors of the difference of two squares	
3.3.2. Use integral exponents on integers and algebraic expressions	1. Operations with radicals: simplification, multiplication and division and addition and subtraction	Show that the diagonal of a square whose side is s can be found by the formula $d = s\sqrt{2}$ [3.3.2.1; 3.3.1.4]
3.3.3. Recognize and identify symmetry and transformations on figures	1. Intuitive notions of line reflection, translation, rotation, and dilation. 2. Line and point symmetry	Provide students with two small mirrors and a small sticker (like a star). Have them set the mirrors upright so they touch on one side and face each other. Have them find the relationship between the angles that the two mirrors form and the number of reiterations of the sticker that they can see. What is the minimum number of stickers that can be seen? What is the maximum number of stickers? How many lines of symmetry are there for each pattern formed? [3.3.3.1; 3.4.1.5]
3.3.4. Use field properties to justify mathematical procedures	1. Distributive and associative field properties as related to the solution of quadratic equations 2. Distributive field property as related to factoring	Identify the field properties used in solving the equation $2(x - 5) + 3 = x + 7$. [3.3.4.1]

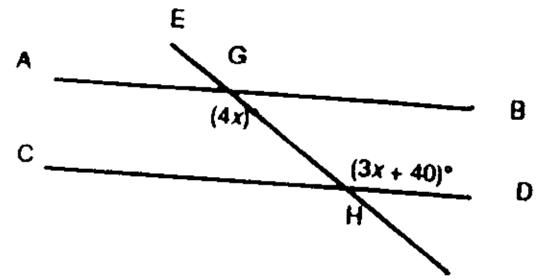
3.4. Modeling/Multiple Representation Math A

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating and connecting mathematical information and relationships.

ASSESSMENT ITEM [3.4.1.]

(1 point)

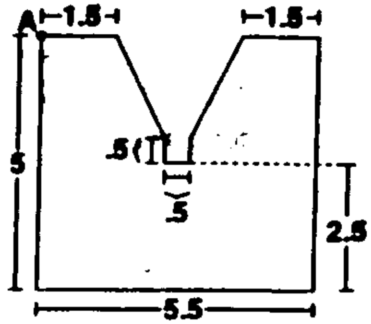
In the diagram below, $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$. Transversal $\overline{EF}$ intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ at G and H, respectively. If m∠ AGH = $4x$ and m∠ GHD = $3x + 40$, what is the value of x ?



(1) 20 (2) 40 (3) 80 (4) 160

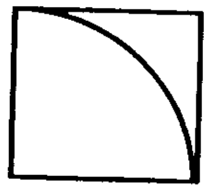
Performance Indicators	Concepts/Skills	Sample Tasks
3.4.1. Represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs	1. Use of variables 2. Formulas and literal equations 3. Undefined terms: point, line and plane 4. Parallel and intersecting lines and perpendicular lines 5. Angles: degree measure, right, acute, obtuse, straight, supplementary, complementary, vertical, alternate interior and corresponding 6. Simple closed curves: polygons and circles 7. Study of triangles: classifications of scalene, isosceles, equilateral, acute, obtuse and right; sum of the measures of angles of a triangle; exterior angle	Shoe sizes and foot sizes are said to be related by the formula $S = 3F - 24$ when S represents the shoe size and F represents the length of the foot in inches. Have students measure their feet and substitute into the formula to see if the formula is accurate for their feet. Have students share their information by gender and note if the formula is reasonable for either men or women's shoe sizes. If not use the student data to try to find a more accurate formula. Is there any relationship between men's shoe sizes and women's shoe sizes? How might you be able to determine this? [3.4.1.1; 3.4.1.2; 3.6.1; 3.7.4.1]

	of a triangle, base angles of an isosceles triangle 8. Study of quadrilaterals: classification and properties of parallelograms, rectangles, rhombus, square and trapezoid 9. Study of solids: classification of prism, rectangular solid, pyramid, right circular cylinder, cone and sphere 10. Sample spaces: list of order pairs of n-tuples, tree diagrams, dot graphs	
3.4.2. Justify the procedures for basic geometric constructions	1. Basic constructions: copy line and angle, bisect line and angle and draw a perpendicular line 2. Comparison of triangles: congruence and similarity 3. Pythagorean Theorem	Explain why the basic construction of bisecting a line is valid. [3.4.2.1]

3.4.3. Use transformations in the coordinate plane	1. Reflection in a line and in a point 2. Translations 3. Dilations	Provide students with a Cartesian Coordinate grid and have them plot the following drawing on it starting point A at (-7,6). On the same grid, in a different color the student is to draw the figure as a reflection in the x-axis and state a rule for making a reflection in the x-axis.  [3.4.3.1; 3.3.3.2]
3.4.4. Develop and apply the concept of basic loci to compound loci	Locus 1. At a fixed distance from a point 2. At a fixed distance from a line 3. Equidistant from two points 4. Equidistant from two parallel lines 5. Equidistant from two intersecting lines 6. Compound locus	High Street and Main Street bound the town park. The two streets are parallel to each other and 100 meters apart. They are also perpendicular to First and Second Streets which are 300 meters apart. The maintenance crew is instructed to plant a tree equidistant from High and Main and 200 meters from the corner of High and First. Make a drawing to show where the tree is to be planted. [3.4.4.4; 3.4.4.5; 3.4.4.63.4.1.4]
3.4.5. Model real-world problems with systems of equations and inequalities.	1. Systems of linear equations and inequalities	A large Ping-Pong set contains 4 paddles and 6 Ping-Pong balls. A small set contains 2 paddles and 1 ball. A store owner receives a "broken shipment" of 100 paddles and 110 balls. Can she divide these evenly into big and small sets? 3.4.5.1; 3.4.1.1; 3.3.1.3]

3.5 Measurement Math A
Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

ASSESSMENT ITEM [3.5.1]
(3 points)
Ms. Brown plans to carpet part of her living room floor. The living room floor is a square, 20 feet by 20 feet. She wants to carpet a quarter-circle as shown below.

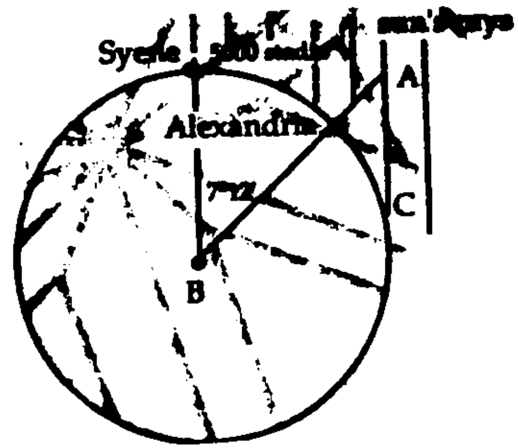


20'

Find, to the *nearest square foot,* what part of the floor will remain uncarpeted. Show how you arrived at your answer.

Performance Indicators	**Concepts/Skills**	**Sample Task**
3.5.1 Apply formulas to find measures such as length, area, volume, weight, time and angle in real-world contexts	1. Perimeter of polygons and circumference of circles 2. Area of polygons and circles 3. Volume of solids	Draw a diagram of a goat pen that will have an area of 800 square meters, using no more than 100 meters of fence wire, if it can be done. If it cannot be done, explain why it cannot. [3.5.1.1; 3.5.1.2]
3.5.2. Choose and apply appropriate units and tools in measurement situations	1. Perimeter of polygons and circumference of circles 2. Area of polygons and circles 3. Volume of solids	While watching a TV detective show you see a crook running out of a bank carrying an attaché case. You deduce from the conversation of the two stars in the show that the robber has stolen **$1** million in small bills. Could this happen? Why or why not? Hint: 1. An average attache case is a rectangular prism (18" x 5" x 13"). 2. You might want to decide the smallest denomination of bill that will work. [3.5.1.3; 3.5.2.3]

3.5.3. Use dimensional analysis techniques	1. Dimensional analysis	The first stage thrust of the Saturn 5 rocket varied between 7,680,000 lb at ignition to 9,115,000 lb just before burnout. The weight of the rocket and the command module at launch was 6,391,120 lb. According to Newton's law the acceleration of an object is equal to the net force in the direction of the acceleration divided by its mass [$a = (F/m)$]. The mass of an object in the English system of units is expressed in terms of the slug, which is the weight of the object in pounds divided by the local acceleration of gravity in ft/s^2, or m - (w/g). (The value of g near the surface of the earth is usually taken as 32.2 ft/s^2.) The net upward force on the unit at ignition would be the thrust of its engines less its weight. What is the acceleration of the unit at ignition? What is this acceleration expressed in g's? [3.5.3.1; 3.3.1.2;3.4.1.2; 4.5.1 (Physical setting)]
3.5.4. Use statistical methods including the measures of central tendency to describe and compare data.	1. Collecting and organizing data: sampling, tally, chart, frequency table, frequency histogram and cumulative frequency histogram 2. Measures of central tendency: mean, median, mode 3. Quartiles and percentiles	Have students also determine their parents' monthly electric charges and find the mode, median and mean monthly charge. If you were trying to budget money each month for electricity which average would be the most useful mode, median or mean and why? [3.5.4.2]
3.5.5. Use trigonometry as a method to measure indirectly	1. Right triangle trigonometry	The altimeter of a Navy reconnaissance plane records 5000 feet as it passes over its carrier. At the same instant a submarine is sighted just under the surface, and its angle of depression from the plane is 25 °. What is the distance from the sub to the carrier? [3.5.5.1; 3.4.1.7]

<table>
<tr>
<td>3.5.6. Apply proportions to scale drawings and direct variation</td>
<td>1. Ratio
2. Proportion
3. Percent
4. Similar figures
5. Similar polygons: ratio of perimeters and areas
6. Direct variation</td>
<td>In 200 BC, Eratosthenes devised this ingenious method for measuring the distance around the earth. To measure the circumference of the Earth Erotosthenes used his knowledge of geometry, particularly the theorem, Parallel lines cut by a transversal form congruent alternate angles. He determined that at noon during the summer solstice in the city of Syene (Egypt) a vertical rod did not cast a shadow, while in Alexandria (500 miles away) the vertical rod cast a shadow that formed a 7°12" angle. With this information, he was able to calculate the circumference of the earth to within 2% of its actual value.


In the drawing above light rays travel parallel to each other, ∠CAB and ∠B in the diagram are congruent alternate interior angles. Thus the distance between Syene and Alexandria is proportional to the distance around the earth as the ratio of the number of degrees in ∠B is to the number of degrees in a circle. What was Eratosthenes' estimate of the circumference of the earth?
[3.5.9.1; 3.5.6.1; 3.5.6.2; 3.5.6.5; 3.5.6.6]</td>
</tr>
</table>

3.5.7. Relate absolute value, distance between two points, and the slope of a line to the coordinate plane	1. Absolute value and length of a line segment 2. Midpoint of a segment 3. Equation of a line: point-slope and slope intercept form	Students in teams record bounce heights of various kinds of balls (like ping pong balls or tennis balls) by dropping them from different heights and noting the height they bounce. Students graph their results on a coordinate plane with the line passing through the origin. The x-axis should be the drop height and the y-axis should be the bounce height. Have students determine the slope of the line and the equation for the line. They should notice that all experiments with the same type of ball produce approximately the same equation and that the slope is also the ratio of the bounce height to the drop height in their experiment. [3.5.7.1; 3.7.1.3;3.7.3.1; 3.7.4.1]
3.5.8. Explain the role of error in measurement and its consequence on subsequent calculations	Error of measurement and its consequences on calculation of 1. Perimeter of polygons and circumference of circles 2. Area of polygons and circles 3. Volume of solids 4. Percent of error in measurements	An odometer is a device that measures how far a bicycle (or a car) travels. Sometimes an odometer is not adjusted accurately, and gives readings which are consistently too high or too low. Paul did an experiment to check his bicycle odometer. He cycled 10 laps around a race track. One lap of the track is exactly 0.4 kilometers long. When he started his odometer read 1945.68 and after the 10 laps his odometer read 1949.88. Compare how far Paul really traveled with what his odometer read. Make a table that shows numbers of laps in multiples of 10 up to 60 laps, the distance Paul really travels and the distance the odometer would say he traveled. Draw a graph to show how the distance shown by the odometer is related to the real distance traveled. Find a rule or formula that Paul can use to change his incorrect odometer readings into accurate distances he has gone from the start of his ride. An odometer measures how far a bicycle travels by counting the number of times the wheel turns around. It then multiplies this number by

		the circumference of the wheel. To do this right the odometer has to be "set" for the right wheel circumference. If it is set for the wrong circumference, it's readings are consistently too high or too low. Before Paul's experiment he estimated that his wheel circumference was 210 cm. Then he set his odometer for this circumference. Use the results of his experiment to find a more accurate estimate for the circumference. [3.5.8.1]
3.5.9. Use geometric relationships in relevant measurement problems involving geometric concepts.	1. Similar polygons: ratio of perimeters and areas 2. Similar figures 3. Comparison of volumes of similar solids	Phil works for a printing company. He has been given the job of ordering boxes to ship dictionaries. The books are 3 inches thick, 6 inches wide, 10 inches long and weigh 4 pounds. He wants to have boxes made which will hold 2 dozen books, with no wasted space in the box. No dimension of the boxes can be greater than 36 inches. What should be the dimensions of the boxes he orders? Can it be done? If not explain why. [3.5.9.3]

3.6 Uncertainty Math A
Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

ASSESSMENT ITEM [3.6.3; 3.6.4.]
(1 point)
Erica cannot remember the correct order of the four digits in her ID number. She does remember that the ID number contains the digits 1,2, 5, and 9. What is the probability that the first three digits of Erica's ID number will all be odd numbers?

(1) ¼ (2) 1/3 (3) ½ (4) 3/4

Performance Indicators	Concepts/ Skills	Sample Tasks
3.6.1. Judge the reasonableness of results obtained from applications in algebra, geometry, trigonometry, probability and statistics	1. Theoretical vs. empirical probability	A box contains 20 slips of paper. Five of the slips are marked with a "X", seven are marked with a "Y", and the rest are blank. The slips are well mixed. Determine the probability that a blank slip will be drawn without looking in the bag on the first draw. Have students determine the probability theoretically and then each conduct the experiment with ten trials and see how close the empirical probability was to the theoretical probability. Combine data from all students in the class to see if a larger number of trials will result in an empirical probability that more closely resembles the theoretical probability. [3.6.1.1;3.6.2.1]
3.6.2. Use experimental and theoretical probability to represent and solve problems involving uncertainty	1. Single and compound events 2. Problems involving AND and OR 3. Probability of the complement of an event	Suppose your landlord allows you to choose from among 6 rental plans. Which would give you the lowest average rent? Explain your answer. Try to design another rental plan so that the average rent is lower than any of the plans below. <u>Rental Plan 1</u> You pay $375 per month <u>Rental Plan 2</u> Each month, you flip a coin Get "heads" and you pay $300. Get "tails" and you pay $400. <u>Rental Plan 3</u> Each month you pick A card, at random from a Standard deck (no jokers) If it is an ace, you pay $600. If it is a face card, you pay $500. Otherwise you just pay $300. <u>Rental Plan 4</u> Each month, the landlord comes and watches you put a $5 bill, a $50 bill, a $100 bill and a $500 bill in a bag. The bag is shaken and then he gets to reach in and pick two bills, at random. What he picks, he

		keeps. **Rental Plan 5** Each month, you get to roll two dice. If your total is 4 or less, you pay $1000. Otherwise, you pay nothing. **Rental Plan 6** Each month, you pick a card, at random, from a Standard deck (no jokers). If it is an ace, you pay $2000. If it is a numbered card (2 through 10), your rent is the number you picked multiplied by 50. If you pick a face card, You pay nothing. [3.6.2.1; 3.6.3.1]
3.6.3. Use the concept of random variable in computing probabilities	1. Mutually exclusive events 2. Counting Principle 3. Sample space 4. Probability distribution 5. Probability of the complement of an event	Two dice are tossed and the sum of the numbers that come up are recorded. What are all the possible sums and what is the probability of each sum. Determine if the requirements of a probability distribution are met in this example. [3.6.3.3; 3.6.3.4; 3.4.1.10]
3.6.4. Determine probabilities using permutations and combinations	1. Counting principle 2. Factorial notation 3. Permutations: ${}_nP_n$ and ${}_nP_r$ 4. Combinations: ${}_nC_n$ and ${}_nC_r$	A home security device has ten buttons. When three different buttons are pushed in the proper sequence the alarm does not go off. No button can be pushed twice. If you forget the correct code, what is the probability that by randomly pushing three of the buttons you will pick the correct code. [3.6.4.3; 3.6.2.2]

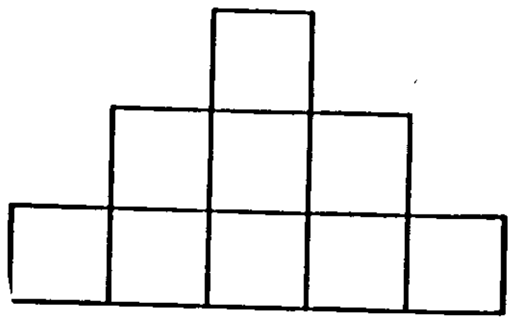
3.7. **Patterns/ Functions Math A**

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

ASSESSMENT ITEM [3.7.4.1]

(2 points)

Haley wishes to build a "tower" out of blocks so that each row has two more blocks than the row above it, as shown in the drawing below.



If Haley begins with 60 blocks, how many complete rows of this tower will she be able to build? Show how you arrived at your answer.

Performance Indicator	**Concepts/ Skills**	**Sample Tasks**
3.7.1. Represent and analyze functions using verbal descriptions, tables, equations and graphs	1. Techniques for solving equations and inequalities 2. Techniques for solving factorable quadratic equations 3. Graphs of linear relations: slope and intercept 4. Graphs of conics: circle and parabola 5. Graphic solution of systems of linear equations, inequalities and quadratic-linear pair 6. Algebraic solution of systems of linear equations, inequalities and quadratic-linear pair by substitution method and	Using a graphing calculator have students investigate how the function $f(x) = ax^2$ changes as the value of **a** changes. What happens as $\| a \|$ increases. What happens when $a > 0$ in contrast to when $a < 0$? Investigate what happens when the graph of the quadratic function $ax^2 + bx + c$ when we vary the coefficients a, b, and c. [3.7.1.4]

	addition-subtraction method	
3.7.2. Apply linear and quadratic functions in the solution of problems	1. Graphic and algebraic solutions of linear and quadratic functions in the solution of problems	The trajectory of a baseball after it leaves the bat can be described by the equation $h(x) = -0.05x^2 + 5.4x$ where $h(x)$ denotes the height of the ball when it has traveled x yards from home plate. Have students graph the function and determine the greatest height reached by the baseball, the horizontal distance of the ball from home plate when it reaches its greatest height and the horizontal distance traveled by the ball. [3.7.1.4; 3.7.2.1; 3.7.4.1]
3.7.3. Translate among the verbal descriptions, tables, equations and graphic forms of functions	1. Translate linear and quadratic functions, systems of equations, inequalities and quadratic linear pairs between representations that are verbal descriptions, tables, equations, or graphs	"Grab bag" assortments of fishing lures all contain $6 worth of lures. Selection A contains three plugs and one jig. Selection B has a plug, two spoons, and a jig. Selection C has five jigs and two spoons. What is the cost of each type of lure? Solve this problem both graphically and algebraically and explain how the solutions can be found on the graph. [3.7.3.1; 3.7.1.4; 3.7.1.5; 3.7.2.1; 3.7.4.1]
3.7.4. Model real-world situations with the appropriate function.	1. Determine and model real-life situations with appropriate functions	In the fact book *The Hidden Game of Baseball*, John Thorn and Pete Palmer present the following formula for determining the probability how many runs a particular player will make. Runs = .46(singles) + .8(doubles) + 1.02(triples) + 1.4 (home runs) + .33(walks + hit-by-pitches) + .3(stolen bases) - .6(caught stealing) -.25(at bats-hits) -.5(outs on base) Have students discuss what this equation means and whether it is reasonable. Why or why not? [3.7.4.1.; 3.7.3; 3.7.1]

MATHEMATICS CORE CURRICULUM GUIDE FOR MATH B

Standard 3: Mathematics

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability and trigonometry.

INTRODUCTION

Problem solving should be a means as well as a goal of instruction. Students should engage in many problem-solving situations and have the opportunity to reflect upon solutions. They should be actively involved individually and in groups in constructing and applying mathematical ideas. Students should engage in a variety of instructional formats such as small groups, individual explorations, peer instruction, whole-class discussions and project work. They should be expected to communicate mathematical ideas in a variety of formats such as drawing diagrams, providing written explanations, using equations, inequalities, graphs, and manipulatives. They should recognize and apply the interrelatedness of mathematical topics and use technology as a tool for learning and doing mathematics. However, the use of technology should not be a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic manipulations. Teachers should use effective questioning techniques that promote student interaction and provide for the systematic maintenance of student learning and embed review in the context of new topics and problem situations.

By the time students are ready to take Math B, they should understand and be able to justify advanced and abstract ideas in algebra, geometry, data analysis, probability, and trigonometry. They should be able to perform complex algebraic simplifications and manipulations as required to solve problems. They should be able to use algebraic, logic, geometric and trigonometric arguments to prove important mathematical ideas. They should have a deep understanding of families of functions, their use in the world and the mathematical techniques required to write, solve, simplify and interpret features of standard functions. Students should understand and apply the connection between a function and its inverse; between right triangle trigonometry and circular functions; and between coordinates in polar, vector and rectangular form. Number systems are expanded to include complex numbers. Transformations are extended to functions. Argument is extended to direct and indirect proof. Data analysis extends to include normal distributions and standard deviation. Probability includes the binomial theorem and Bernoulli experiments. Functions extend to include exponential and logarithmic functions.

Manipulatives continue to be a crucial element of instruction especially when new concepts are being introduced. Some useful manipulatives for this level include miras (or mirrors), probability tools like spinners and dice, cones that can be sliced to show conic sections, geoboards, tangrams, tesselation tiles, pattern blocks and PentaBlocks. Calculators should be a minimum of graphing and can be useful for developing conceptual understanding for topics such as successive approximations, graphical representations for solving equations and inequalities, trigonometric, algebraic, exponential and logarithmic graphs and problem solutions, statistics and probability problems. In addition, computers can be useful for exploring 2-D and 3-D figures.

Assessment of learning should be an integral part of instruction to inform the teacher of the effectiveness of instruction as well as student progress. Students should be expected to express their understanding with graphs, symbols and with written and oral descriptions. Class discussions should not be confined to recall questions but should evolve around open-ended questions that get students to explain their ideas.

The performance indicators, concept/skills, sample tasks and assessment items that follow are suggestions and meant to be a guide for use by local education agencies in developing their own curriculum. The order and placement of topics does not imply an expectation of how classroom instruction should be sequenced. They are not meant to be restrictive. We have used the following sources for ideas for sample tasks and assessment items. Teachers may wish to refer to them form more ideas.

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3.1 Mathematical Reasoning

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Performance Indicators	Concepts/Skills	Sample Tasks
3.1.1 Construct simple logical arguments.	1. Laws of Reasoning. ♦ Laws of Detachment ♦ Law of Contrapositive ♦ Law of Disjunctive Inference ♦ Law Syllogism ♦ DeMorgan's Law	Prove whether the following argument is valid or invalid. Write a complete explanation of your steps of proof. If squares are circles, then rectangles are ellipses. If triangles are cones, then circles are spheres. Either squares are not circles, or triangles are not cones. Therefore, wither circles are not spheres, of rectangles are not ellipses. [3.1.1.1; 3.1.2.1; 3.1.2.2]
3.1.2 Follow and judge validity of logical arguments.	1. Use rules including Laws of Reasoning to determine the truth values of compound statements and proving tautologies. 2. Conditional-converse, inverse and contrapostive.	We know that any argument can be written in the form of an "if-then" sentence. Now, since $P \rightarrow Q$ is true if P is false or if Q is true, why isn't an argument valid if we can get either a false premise or a true conclusion? (For example: "If something is an animal, then it's a tiger. This is a tiger. So it's an animal." Now obviously the first premise is false. So how come the argument isn't valid, since we said we could treat it as an "if-then" sentence?) [3.1.2.2}
3.1.3 Use symbolic logic in the construction of valid arguments.	1. Develop direct and indirect logic proofs.	See example in 3.1.1
3.1.4 Construct proofs based on deductive reasoning.	1. Euclidean, analytic, logical, and trigonometric direct proofs	Assign students to groups of about 6. Have them write a proof and put each step on a separate piece of paper. They give their strips to the members of another group. If there are not enough people in the other group for all the proof's steps some students can receive more than

		one step. If there are not enough steps to the proof for each student in the group, extra "clues" can be given as "wild cards".
3.1.5 Construct indirect proofs.	1. Euclidean, analytic, logical and trigonometric indirect proofs.	Prove that $\sqrt{2}$ is an irrational number. [3.1.5.1; 3.2.1.; 3.2.3]

Assessment Item [3.1.4]

Prove that if two secants are drawn to a circle from an external point, the product of the lengths of one secant and its external segment is equal to the product of the lengths of the other secant and its external segment.

3.2 Number and Numeration

Students use number sense and numeration to develop an understanding of multiple uses of numbers in real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

Performance Indicators	Concepts/Skills	Sample Tasks
3.2.1 Understand and use rational and irrational numbers.	1. Determine from the discriminate of a quadratic equation whether the roots are rational or irrational. 2. Rationalize denominators. 3. Simplifying of algebraic fractions with polynomial denominators. 4. Factor out common monomials and factor the resulting polynomial by factoring polynomials and the difference in two squares. 5. Simplify complex fractions.	Physicists tell us that the altitude h in feet of a projectile t seconds after firing is $h = -16t^2 + v_u t + h_0$ where v_u is the <u>upward</u> component of its initial velocity in feet per second and h_0 is the altitude in feet from which it is fired. A rocket is launched from a hilltop 2400 feet above the desert with an initial upward velocity of 400 feet per second. When will it land on the desert? Discuss what the discriminate can tell you about the solution to this problem. Then use the quadratic equation to find the solution and explain you answer. [3.2.1.1; 3.2.4.3]
3.2.2 Recognize the order of the	1. Give rational approximations of	Explain the difference between the numbers in each

real numbers.	irrational numbers to a specific degree of accuracy.	set below and arrange the numbers in each set in order from lowest to highest. ◆ $4.\overline{87}$, $4.8\overline{7}$, 4.8, 4.87, $4.\overline{8}$ ◆ $\overline{2.367}$, $2.\overline{367}$, $2.3\overline{67}$, $2.36\overline{7}$, 2.367 [3.2.2.1; 3.2.1]
3.2.3 Apply the properties of the real numbers to various subsets of numbers.	1. Determine which set of reals and usual subsets meet the requirements of a field. 2. Use the properties of real numbers in the development of algebraic skills.	Indicate whether each statement below is true or false and for each false statement find a real number replacement for a, b, and c which will illustrate its falseness. ◆ $(a + b) + c = a + (b + c)$ ◆ $(a - b) - c = a - (b - c)$ ◆ $(ab)c = a(bc)$ ◆ $(a \div b) \div c = a \div (b \div c)$ [3.2.3.1]
3.2.4 Recognize the hierarchy of the complex number system.	1. Subsets of complex numbers 2. Verify that the set of complex numbers form a field under the operations of addition and multiplication. 3. Determine from the discriminate of a quadratic equation whether the roots are imaginary, rational or irrational.	Indicate whether each statement below is true and explain why using mathematical language. ◆ All natural numbers are integers ◆ All real numbers are irrational ◆ All natural numbers are rational numbers [3.2.4.2]
3.2.5 Model the structure of the complex number system.	1. Imaginary unit of complex numbers. 2. Standard form of complex numbers.	Show that when the real number c is written as a complex number $c + 0I$, we have $(c + 0i)(a + bi) = ca + cbi$; that is, $c(a + bi) = ca + cbi$. [3.2.5.1; 3.2.4.2]

Assessment Item [3.2.4]

Show that complex numbers together with the operations of addition and multiplication satisfy the six field properties.

3.3 Operations

Students use mathematical operations and relationships among then to understand mathematics.

Performance Indicators	Concepts/Skills	Sample Tasks
3.3.1 Use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions.	1. Laws of exponents. 2. Complete operations on fractions with polynomial denominators. 3. Add and subtract rational fractions with monomial and binomial denominators.	A googol is a 1 followed by 100 zeros, and a googolplex is a 1 followed by a googol of zeros. Express these two numbers as powers of 10. [3.3.3.1]
3.3.2 Develop an understanding of and use the composition of functions and transformations.	1. Understand the general concept and symbolism of the composition of transformations. 2. Apply the composition of transformations (line reflections, rotations, translations, glide reflections). 3. Identify graphs that are symmetric with respect to the axes or origin. 4. Isometries (direct, opposite). 5. Applications to graphing (inverse functions, symmetry). 6. Define and compute compositions of functions. 7. Functions (inverse, exponential, logarithmic)	The Environmental Protection Agency has determined that in a certain section of the country the average level of air pollution is $0.5\sqrt{P} + 10,000$ parts per million (ppm), where P is the population. The 1980 census predicts that the population *t* years after 1980 will be 7ooo + $40t^2$. A - Express the pollution level *t* years after 1980 as a composite function and reduce the composite function to a function of *t*. B - What pollution level can be expected in 1990? 2000? [3.3.2.6]
3.3.3 Use transformations on figures and functions in the coordinate plane.	1. Apply transformations (line reflection, point reflection, rotation, translation and dilation) on figures and functions in the coordinate plane. 2. Use slope and midpoint to demonstrate transformations. 3. Use the ideas of transformations to	On graph paper, set up a coordinate system for each figure and graph the figure by plotting coordinates and connecting adjacent vertices. Sketch the reflection of each shape over the line y=x. a. (3,2), (-1,-4), (7,2), (-2,3) b. (1,7), (4,5), (6,-1)

	investigate relationships of two circles. 4. Use translation and reflection to investigate the parabola. 5. Absolute value of complex numbers.	c. (-2,-4), (-1,5), (3,3) d. (6,4), (-2,5), (-2,-2), (3,5), (4, 2.5) What is the relationship between the coordinates of the original figure and its reflection image? State your conjecture in if-then form. Write an argument that you could use with a friend to convince them that your conjecture is correct.
3.3.4 Use rational exponents on real numbers and all operations on complex numbers.	1. Evaluate expressions with fractional exponents. 2. Basic operations of complex numbers 3. Simplify square roots with negative radicands. 4. Use the product of a complex number and its conjugate to express the quotient of two complex numbers. 5. Cyclic nature of the powers of i. 6. Solving quadratic equations.	Fractals are created from iterations of the same function and produce interesting graphs. Let students use a graphing calculator to explore the orbits of the following functions and describe any patterns they notice. ◆ $k(x) = (x + 9/x)/2$ ◆ $m(x) = (x = 2/x)/2$ [3.3.4.2]
3.3.5 Combine functions using the basic operations and the composition of two functions.	1. Determine the value of compound functions. 2. Pairs of equations	*Cost Analysis:* The cost C to produce x units of a given product per month is given by $C = f(x) = 19{,}200 + 160x$ If the demand x each month at a selling price of \$$p$ per unit is given by $X = g(p) = 200 - p/4$ Find $(f \bullet g)(p)$ and interpret. [3.3.5.1]

Assessment item [3.3.4]

The expression i^{35} is equal to:
1. 1 2. -1 3. i 4. -i

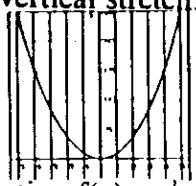
3.4 Modeling/Multiple Representation

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

Performance Indicators	Concepts/Skills	Sample Tasks
3.4.1 Represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs.	1. Express quadratic, circular, exponential and logarithmic functions in problem situations algebraically 2. Use symbolic form to represent an explicit rule for a sequence 3. Relate algebraic expressions to the graphs of functions 4. Definition and graph of an inverse variation (hyperbola).	Draw the graph y=48/x. Make a table using some integral values of x from x=-16 to x=16 ($x \neq 0$). Identify the graph. [3.4.1.4]
3.4.2 Manipulate symbolic representations to explore concepts at an abstract level.	1. Use positive, negative and zero exponents and be familiar with the laws used in working with expressions containing exponents. 2. In the development of the use of exponents, the students should review scientific notation and its use in expressing very large or very small numbers. 3. Rewrite the equality $\log_b a = c$ as $a = b^c$. 4. Solve equations using logrithmic expressions 5. Use the laws for performing calculations of exponents and logarithms	Prove: If x and n are real numbers, and $x > 0$, then $\log_a (x^n) = n\log_a x$ $a > 0$, $a \neq 1$ [3.4.2.5]

	6. Compound functions	
3.4.3 Choose appropriate representations to facilitate the solving of a problem.	1. Select between exponential and logarithmic forms of equations the most efficient.	In April 1986 there was a major nuclear accident at the Chernobyl power plant in the Soviet Union. The atmosphere was contaminated with quantities of radioactive iodine-131, which has a half life of 8.1 days. How long did it take for the level of radiation to reduce to 1% of the level immediately after the accident? Express as both exponents and logs and solve with whichever is easier. [3.4.3.1; 3.4.2.4; 3.4.2.5]
3.4.4 Develop meaning for basic conic sections.	1. Circles 2. Parabolas 3. Hyperbolas 4. Ellipses	Have students find real-world examples of each conic section. [3.4.4.1; 3.4.4.2; 3.4.4.3; 3.4.4.4]
3.4.5 Model real-world problems with systems of equations and inequalities.	1. Solve systems of equations of linear-quadratic systems. 2. Fractional equations 3. Equations with radicals 4. Linear inequalities 5. Absolute value inequalities 6. Quadratic inequalities	Two toy rockets are launched, one ten seconds after the other. The height in feet of the first rocket after $0 \le t \le 16$ seconds is given by $h(t) = -16t^2 + 256t$. The height of the second one after $10 \le t \le 20$ seconds is given by $g(t) = -16t^2 + 480t - 3200$. How many seconds after the first rocket is launched are the rockets at the same height? [3.4.5.1]
3.4.6 Model vector quantities both algebraically and geometrically.	1. The Law of Sines and Cosines can be used with a wide variety of problems involving triangles, parallelograms and other geometric figures in applications involving the resolution of forces both algebraically and geometrically	The lift of an airplane wing is 750 lb. The drag is 300 lb. What is the magnitude and the direction of the resulting force? Draw a picture of the wing showing the lift and drag forces. Represent the problem geometrically and find the resultant for algebraically. [3.4.6.1; 3.4.6.8.2]
3.4.7 Represent graphically the sum and difference of two	1. Represent the basic operations of addition and multiplication and their	Have students investigate whether or not the difference of two complex conjugates can be a

complex numbers.	conjugates.	real number. [3.4.7.1]
3.4.8 Model and solve problems that involve absolute value and vectors.	1. Solve using the polar form of complex numbers. 2. See 3.4.6.1.	The amount of alternating current in a circuit is restricted by the impedance *Z* (in ohms). If a resistor, inductor, and capacitor are connected in series the impedance is $Z = R + j(X_L - X_C)$, with magnitude $\lvert Z \rvert$ and direction Θ given, respectively, by $\lvert Z \rvert = \sqrt{R^2 + (X_L - X_C)^2}$ and $\theta = \tan^{-1}((X_L - X_C)/R)$ Provide students with various values for *R*, X_L *and* X_C and have them compute the magnitude and direction of *Z* (to the nearest whole number) for each set of data. [3.4.9.1; 3.4.6.1]
3.4.9 Model quadratic inequalities both algebraically and graphically.	1. Model algebraically and graphically inequalities such as $x^2 - 5x - 6 < 0$ to find the possible solutions.	See example in 3.4.5
3.4.10 Model the composition of transformations.	1. The composition of two line reflections when the two lines are parallel 2. The composition of two rotations about the same center. 3. The composition of two translations. 4. The composition of a line reflection and a translation in a direction parallel to the line of refection (glide refection)	Give students cut out triangles. Have them draw a line and put a point on it for a vertex (straight angle) by doing translations with the triangle, students are to show that the sum of the measures of the angles of a triangle is 180°. Have students list their translations in order (The translation for the first two angles can be done with a slide. The third angle can be done with a composition of line rotation and slide. Have students prove that the translations are legitimate using rules of transformations and parallel lines. [3.4.10.1; 3.4.10.3]
3.4.11 Determine the effects of changing parameters of the graphs	1. Be able to sketch the effects of changing the value of a in the function	The graph of a function can be transformed in a number of ways. We will consider three: Vertical

<table>
<tr>
<td>of functions.</td>
<td>y = a^x Characteristics to be emphasized are-
◆ The domain of an exponential function is the set of real numbers
◆ The range of an exponential function is the set of positive numbers
◆ The graph of any exponential function will contain the point (0, 1)
◆ The exponential function is one-to-one
◆ If a > 1, the graph rises, but if 0 < 1 < 1, the graph falls
◆ The graphs of y = a^x and y = a^{-x}, a > 0, and a ≠ 1, are reflection of each other on the y-axis
2. The logarithmic function as the inverse of the exponential function with the following characteristics
◆ Since the exponential function is one-to-one, its inverse, the logarithmic function, exists
◆ The domain of the logarithmic function is the set of positive real numbers
◆ The range of the logarithmic function is the set of all real numbers
◆ The graph of any logarithmic function will contain the point (1,))
◆ The graphs of y = a^x and x = a^y, a >0, and a ≠ 1 are reflections of each other in the line y = x.</td>
<td>shift, horizontal shift, and vertical stretch. The function we will use Is f(x)=x^2. The graph of this function is shown to the right.

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<tr><td>-3</td><td>9</td><td>18</td><td>6</td><td>36</td></tr>
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Use the table to graph f(x)=$2x^2$, g(x)=x^2-3, and h(x)=$(x-3)^2$ on the coordinate axis.

Compare each graph you drew to the graph of f(x)=x^2.
-Which function has a graph that is a vertical shift of the graph of f(x)=x^2? Is the shift upward or downward?
-Which function has a graph that is a horizontal shift of the graph of f(x)=x^2? Is the shift right or left?
-Which function has a graph that is a vertical stretch of the graph of f(x)=x^2?

Instead of being stretched vertically, a graph can be shrunk. What function would have a graph that is a vertical shrink of the graph of f(x)=x^2 by a factor of ½? [3.4.11.1]</td>
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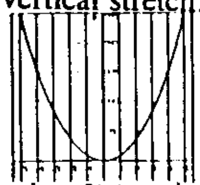
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	♦ Sine ♦ Cosine ♦ Tangent ♦ Reciprocal trigonometric functions 2. Relate reference angles, amplitude and period to the solution of real-world problems	an oscilloscope (an instrument in which the variations in a fluctuating electrical quantity appear temporarily as a visible wave from on the fluorescent screen of a cathode-ray tube). Have students analyze and display the tones produced by the different instruments [3.4.14.2]

Assessment item [3.4.1; 3.4.2]

Given: $f = \{(x,y) \mid y = \log_2 x\}$

a. On graph paper, sketch and label the graph of the function f. [3]
b. Write a mathematical explanation of how to form the inverse of function f. [2]
c. On the same set of axes, sketch and label the graph of the function f^{-1}, the inverse of f. [3]
d. Write an equation for f^{-1}. [2]

3.5 Measurement

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

Performance Indicators	Concepts/Skills	Sample Tasks
3.5.1 Use trigonometry as a method to measure indirectly.	1. Triangle Solutions 2. Right triangle trigonometry 3. Unit circle 4. Angle rotation - the measure of an angle can be a real number. 5. See 3.4.6	Use the sine function to find the area of a right triangle (area equals one-half of the product of any two sides and the sine of the included angle). [3.5.1.2]
3.5.2 Understand error in measure and its consequence on subsequent calculations.	1. Error of measurement of angles and length of the sides of a triangle and its consequence to the solution of trigonometric problems.	The pitch of a roof is defined as the vertical rise corresponding to a given horizontal run. Thus a 4-12 pitch indicates a vertical rise of 4 feet for each horizontal

		advance of 12 feet. What angle does the roofline make with the horizontal in this case? [3.5.2.1]
3.5.3 Derive and apply formulas relating angle measure and arc degree measure in a circle.	1. Express angle measure in terms of degrees and radians. 2. Reference and coterminal angles 3. Understand the derivation and apply formulas for sine, cosine, tangent and their reciprocal trigonometric function. 4. Sum and difference of two angles 5. Double and half angles. 6. Vectors	Give students a cone-shaped drinking cup. Have the students cut the side from the brim to the apex of the cone and flatten out the cup. The shape of the flattened surface will be a circle with a sector missing. Ask them to use the shape and the ideas of unit circles to help them find the surface area of the cone. [3.5.3.1; 3.5.3.2; 3.5.3.3; 3.5.3.4]
3.5.4 Prove and apply theorems related to lengths of segments in a circle.	1. Prove and apply theorems related to arcs, chords, tangents, secants and angles. 2. Prove theorems related to congruence.	Prove that any trapezoid inscribed in a circle is an isosceles trapezoid; that is, at least one pair of opposite sides are equal. [3.5.4.1; 3.5.4.2; 3.1.4.1]
3.5.5 Define the trigonometric functions in terms of the unit circle.	1. Sine, cosine, tangent and their reciprocal functions on the unit circle. 2. Radian measure. 3. Coordinates of a point on the unit circle expressed as $\cos\theta$ $\sin\theta$. 4. Special angles of 30°, 45° and 60°. 5. Reference angles. 6. Amplitude and period. 7. Reflections in the line $y = x$. 8. Inverse functions.	Sketch the six basic trigonometric functions and their inverses on the graphing calculator. Superimpose the functions with their inverses on a graphing calculator. [3.5.5.1; 3.5.5.6; 3.5.5.8]
3.5.6 Relate trigonometric relationships to the area of a triangle and to general solutions of triangles.	1. Application of the sine function in the solution of the area of a triangle. 2. Law of sines ◆ Finding a side given ASA or Aas	Prove that if ABC is a right triangle, the Law of Cosines reduces to the Pythagorean theorem. [3.5.6.3;3.1.4.1]

	♦ The ambiguous case (SSA) ♦ Finding a side given SSA 3. Law of cosines ♦ Finding a side given SAS ♦ Finding an angles given SSS	
3.5.7 Apply the normal curve and its properties to familiar contexts.	1. Intuitive use of the normal curve in real world situations. 2. Mean on the bell curve 3. Standard deviation	As one of its administration criteria, a college requires an SAT math score that is among the top 70% of all scores. The mean score on the math portion of the SAT is 500 and the standard deviation is 100. What is the minimum acceptable score and justify your answer by drawing a sketch of the normal distribution and shading the region representing acceptable scores. [3.5.7; 3.6.8]

Assessment item [3.5.1; 3.5.6; 3.4.6; 3.4.8]

Two forces of 20 Newtons and 10 Newtons act at an angle of 60°30′ to each other. Find the magnitude of the resultant, to the nearest Newton.

3.6 Uncertainty

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

Performance Indicators	Concepts/Skills	Sample Tasks
3.6.1 Judge the reasonableness of results obtained from applications in algebra, geometry, trigonometry, probability, and statistics.	1. Find the solution of a quadratic equation both algebraically and graphically as a check 2. Substitution as a check for solutions to	Use substitution for a check for solutions to equations and inequalities. [3.6.1.2]

	equations and inequalities 3. Check solutions of permutation problems using combination formulas as a check and vice versa 4. Using proof as a check on the validity of geometric constructions 5. Compare histograms with formula derived solutions for mean, median, variation and standard deviation 6. Use the Quotient Identities, Reciprocal Identities, and the Pythagorean Identities to establish that other equations are identities	
3.6.2 Judge the reasonableness of a graph produced by a calculator or computer.	1. Determine the effects of changing the parameters of graphs of linear, quadratic, trigonometric, exponential, and circular functions 2. See 3.4.11.1 3. See 3.4.11.2	A rich philanthropist, who loved mathematics, agreed to sponsor an 18 hole golf tournament at the local country club. In order to enter, a contestant had to pay 2 cents and select either a linear, quadratic, or exponential formula to calculate how many CENTS he/she would receive for a winning hole. In each of the following formulas, X represents the number of the winning hole. linear, $Y=2X$; quadratic, $Y=X^2$, exponential, $Y=2^X$. Why bother entering if the payoff is in pennies? Use your graphing calculator to investigate. Describe numerically how the amounts change from one hole to the next for each method. Which method would you select on your entry from and why?

		[3.6.2.1]
	1. Review concepts from Math A. See Math A 3.6.2.	See Math A
3.6.4 Use the concept of random variable in computing probabilities.	1. Review concepts from Math A. See Math A 3.6.3	See Math A
3.6.5 Determine probabilities using permutations and combinations.	1. Review concepts from Math A. See Math A 3.6.4.	See Math A
3.6.6 Interpret probabilities in real-world situations.	1. Applications of the probability of exactly r successes in n trials of a Bernoulli experiments. 2. Simple applications of the Binomial Theorem	The principal of the local high school was willing to participate in the school fair dunking booth in which students who paid $1 could push a button that operated a light over the booth which was programmed to flash either red or green. If the light flashed green the principal would fall into the water. If it flashed red, he would not. He was told that the light was set to flash either red or green randomly with a 50% chance of turning green. As it turned out, the principal seemed to be dunked more than 50% of the time. In the first 20 pushes of the button, he was dunked 15 times. He was getting suspicious that probability had been misrepresented to him. Based on the results so far, do you think the principal has justification for being suspicious? What is your reasoning? If you do not think the principal is justified in his suspicions, how many occurrences of 75% dunks would it take to convince you that the light was not set

		at 50% green? If you think the principal is justified in being suspicious what is the smallest occurrences of 75% that would be required to convince you? [3.6.6.2; 3.6.3; 3.6.4; 3.6.5]
3.6.7 Use a Bernoulli experiment to determine probabilities for experiments with exactly two outcomes.	1. Definition of a Bernoulli experiment 2. Case where r successes are assumed to occur first 3. General case	If the problem can be regarded as a Bernolli experiment, state the values of *n*, *p*, *q*, and *r*, and give the answer in symbolic form. If the problem cannot be regarded as a Bernoulli experiment, explain why. ◆ Four balls are drawn with replacement from an urn containing 4 red balls and 2 white balls. What is the probability of drawing exactly 2 red balls? ◆ Four balls are drawn without replacement from an urn containing 4 red balls and 2 white balls. What is the probability of drawing exactly 2 red balls? [3.6.6.1; 3.6.7.1]
3.6.8 Apply the concept of random variable to generate and interpret probability distributions.	1. Normal Distribution 2. Binomial Theorem (Pascal's Triangle)	The cause of death is related to heart disease in 52% of the cases studied. Find the probability that in 500 randomly selected cases, the number of heart-disease-related deaths differs from the mean by more than two standard deviations. [3.6.8; 3.5.7]
3.6.9 Create and interpret applications of discrete and continuous probability	1. Measures of central tendency 2. Use of Σ-notation	Which would you expect to have a higher variance: The IQ scores of a class if 25

distributions.	3. Measures of dispersion ◆ Range ◆ Mean absolute deviation ◆ Variance ◆ Standard deviation	statistics students or the IQ scores of 25 randomly selected adults? Use mathematical language to justify your answer. [3.6.9.3; 3.6.8.1]
3.6.10 Make predictions based on interpolations and extrapolations from data.	1. Domain and range 2. Interpolate and extrapolate from graphs of linear, quadratic, trigonometric, circular, exponential and logarithmic function	The boiling point of water is a function of altitude. The table shows the boiling points at different altitudes. Location / Altitude h metres / Boiling point of water, t°C Halifax, NS / 0 / 100 Banff, Alberta / 1383 / 95 Quito, Ecuador / 2850 / 90 Mt. Logan / 5951 / 80 - Graph the relation between the altitude and the boiling point. - Use the graph to estimate the boiling point of water at: a) Lhasa, Tibet, altitude 3680m b) the summit of the earth's highest mountain, Mt. Everest, 8848m.

Assessment item [3.6.3]

In a computer baseball game the probability that a particular baseball player gets a hit is 0.250 or 1/4. In a simulated game that player comes to bat four times. What is the probability that thus player gets at least one hit?

3.7 Patterns/Functions

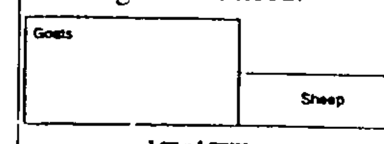
Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

Performance Indicator	Concepts/Skills	Sample Tasks
3.7.1 Use function vocabulary and notation.	1. Definition of a relation 2. See 3.3.2.6 3. Determining if a relation is a function 4. Definition of inverse function 5. Notation for absolute value, compound functions 6. Expressing exponential functions as logs	See examp0le in 3.3.2
3.7.2 Represent and analyze functions using verbal descriptions, tables, equations, and graphs.	See 3.4.1.3;3.4.11; 3.4.13	See example in 3.4.1
3.7.3 Translate among the verbal descriptions, tables, equations, and graphic forms of functions.	See 3.4.3; 3.4.11; 3.4.13	See example in 3.4.13
3.7.4 Analyze the effect of parametric changes on the graphs of functions.	See 3.4.11; 3.4.13; 3.6.2.1	See example in 3.6.2.
3.7.5 Apply linear, exponential, and quadratic functions in the solution of problems.	See 3.4.12	See example in 3.4.12
3.7.6 Apply and interpret transformations to functions.	See 3.3.3	See example in 3.3.3
3.7.7 Model real-world situations with the appropriate function.	1. Characteristics of linear, quadratic, trigonometric, circular, exponential and logarithmic function.	Nita Pass is about to study for a mathematics exam. Nita knows that the test grade is a function of a number of hours studied and knows from past experience that 1 hour of studying will result in a grade of 60; 2 hours, in a grade of 74; and 7 hours in a grade of 84. 1-Show Nita that the grade is not a linear function of the number of hours studied. 2-Assume that the grade varies quadratically with the number of hours studied. Find the equation for the function, draw the graph (show important features: vertex and intercepts).

		3-What is the highest grade Nita could earn? How many hours of study would this require? 4-What is the minimum amount of study time needed to pass the test if the lowest passing grade is 70? What is the grade-intercept and what does it represent in the real world? 5-The quadratic model predicts that Nita could earn zero points on the test. What might happed in the real world that could actually cause her to score zero by studying this ling? 6-Use the grapg to show that there is no real value of time for which the grade will be 100. [3.7.7.1]
3.7.8 Apply axiomatic structure to algebra and geometry.	See 3.1.4; 3.1.5, 3.6.1.4	See example in 3.1.4
3.7.9 Solve equations with complex roots using a variety of algebraic and graphical methods with appropriate tools.	1. Nature of the roots of a quadratic See 3.2.4; 3.3.4; 3.4.7	See example in 3.2.1
3.7.10 Evaluate and form the composition of functions.	See 3.3.5.1, 3.4.2.6	See example in 3.3.5
3.7.11 Solve equations using fractions, absolute values, and radicals.	See 3.3.4.6; 3.4.5;3.4.8	See example in 3.3.4
3.7.12 Use basic transformations to demonstrate similarity and congruence of figures.	1. Translations that provide congruence ♦ Direct Isometries ♦ Opposite Isometries 2. Translations that provide similarity ♦ Dilation	Provide students with examples of Escher prints and have them identify two congruent shapes and the isometries that provide the congruence. [3.7.12.1.; 3.4.10]
3.7.13 Identify and differentiate between direct and indirect isometries.	See 3.7.12	See example 3.7.12
3.7.14 Analyze inverse functions using transformations.	1. Identify inverse functions which are reflections in the line $y = x$.	Graph each relation, its inverse, and $y=x$ on the same coordinate system. Identify all functions in addition to $y=x$. (Use the graph of the original relation to find the graph of the inverse.)

		g: $y=2x-2$ f: $y=-1/2x+2$ p: $y=x^2+1$ q: $y=(x+2)^2$ [3.7.14.1]
3.7.15 Apply the ideas of symmetries in sketching and analyzing graphs of functions.	1. Simplify the graphing of functions by using symmetries with respect to an axis, the origin, or some other point.	Find, if possible, a line of symmetry of the graph of each equation. If there is no line of symmetry, write none. $y=x^2+5$ $y=x^2+4x+1$ $y=x$ [3.7.15.1]
3.7.16 Use the normal curve to answer questions about data.	1. Standard deviation for grouped data See 3.6.9	Find the variance and standard deviation for the data given in the following table. **Frequency table of time (in seconds) it takes the victim of a crime to call the police.** Time — Frequency 0-59 — 1 60-119 — 2 120-179 — 5 180-239 — 14 240-299 — 7 300-359 — 12 360-419 — 9 [3.7.16.1, 3.6.9.3, 3.5.7.3]
3.7.17 Develop methods to solve trigonometric equations and verify trigonometric functions.	1. Proving trigonometric identities 2. Solve first degree trigonometric equations 3. Solve quadratic trigonometric equations 4. Functions of sums and differences of angles 5. Double and half angle formulas	Let groups of students apply the following procedure to construct their own identities. Then they can prove each other's identities. 1. Start with a statement that is always true such as 1 = 1. 2. Replace part(s) of the beginning statement with an equivalent trigonometric expression such as $\sin^2\Theta$

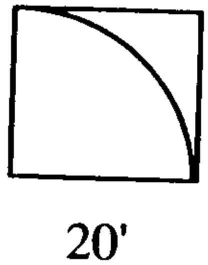
	of a triangle, base angles of an isosceles triangle 8. Study of quadrilaterals: classification and properties of parallelograms, rectangles, rhombus, square and trapezoid 9. Study of solids: classification of prism, rectangular solid, pyramid, right circular cylinder, cone and sphere 10. Sample spaces: list of order pairs of n-tuples, tree diagrams, dot graphs	
3.4.2. Justify the procedures for basic geometric constructions	1. Basic constructions: copy line and angle, bisect line and angle and draw a perpendicular line 2. Comparison of triangles: congruence and similarity 3. Pythagorean Theorem	Explain why the basic construction of bisecting a line is valid. [3.4.2.1]

PERFORMANCE INDICATORS	CONCEPTS/SKILLS	SAMPLE TASKS	ASSESSMENT ITEMS
3.5.1Understand that measurement is approximate, never exact.	1. Identify appropriate metric units for measuring the area, mass, perimeter, volume of a variety of objects.	Have students work in groups to create their own measurement scale (hand, book, string). Determine the distance of certain set locations and objects in the school using their measure and standard measure. [3.5.1, 3.6.1]	Farmer Brown is building
3.5.2 Select appropriate standard and nonstandard measurement tools in measurement activities.	1. Identify equivalent measures within a system of measure. 2. Relate fraction concepts to measurement tools 3. Relate the clock to circle construction and fractions.	Remove all the labels from a variety of cans (different sizes, weights and shapes). Have students estimate the metric weight of each can, and by using both the digital scale and traditional scale, weigh each to see if their estimations were reasonable. [3.5.2.1, 3.5.2.2, 3.5.4.1]	pens for his animals. He wants the goats and the sheep to be in separate areas. He drew the diagram
3.5.3 Understand the attributes of area, length, capacity, weight, volume, time, temperature, and angles.	1. Study time to the 5 minutes, 1 minute and second intervals. 2. Find the area and volume of specific figures, initially by counting units and then extending the counting of units into work with factors and multiplication. 3. Develop and solve measurement problems related to other topic areas.	It's Saturday and you're going to meet your friends for lunch and a movie. You have to leave your home at 11:30 AM. Your parents say you can't go until you finish your work. Your work includes: 40 minutes of math homework 30 minutes to clean your room 15 minutes to fold the laundry 3 Minutes to take out the garbage 60 minutes to eat and get ready to go A. At what time should you get started doing your work? Show all your work. B. Describe how you would use your time between when you wake up and when you leave at 11:30 AM. [3.1.4, 3.5.3.1]	below (reduced 75%) to help him decide how much fencing he will need.  Use a centimeter ruler to measure the diagram. Then label the diagram with your

3.5 Measurement Math A

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

ASSESSMENT ITEM [3.5.1]
(3 points)
Ms. Brown plans to carpet part of her living room floor. The living room floor is a square, 20 feet by 20 feet. She wants to carpet a quarter-circle as shown below.

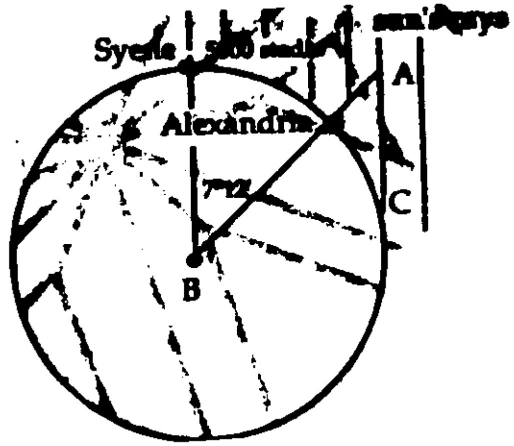


Find, to the *nearest square foot,* what part of the floor will remain uncarpeted. Show how you arrived at your answer.

Performance Indicators	**Concepts/Skills**	**Sample Task**
3.5.1 Apply formulas to find measures such as length, area, volume, weight, time and angle in real-world contexts	1. Perimeter of polygons and circumference of circles 2. Area of polygons and circles 3. Volume of solids	Draw a diagram of a goat pen that will have an area of 800 square meters, using no more than 100 meters of fence wire, if it can be done. If it cannot be done, explain why it cannot. [3.5.1.1; 3.5.1.2]
3.5.2. Choose and apply appropriate units and tools in measurement situations	1. Perimeter of polygons and circumference of circles 2. Area of polygons and circles 3. Volume of solids	While watching a TV detective show you see a crook running out of a bank carrying an attaché case. You deduce from the conversation of the two stars in the show that the robber has stolen $1 million in small bills. Could this happen? Why or why not? Hint: 1. An average attache case is a rectangular prism (18" x 5" x 13"). 2. You might want to decide the smallest denomination of bill that will work. [3.5.1.3; 3.5.2.3]

PERFORMANCE INDICATORS	CONCEPTS/SKILLS	SAMPLE TASKS	ASSESSMENT ITEMS
3.6.1 Make estimates to compare to actual results of both formal and informal measurement.	1. Round numbers using number lines and measuring instruments (meter stick, thermometer).	Give the children a list of small objects in the room (pencil, book, dominoes, straws). Have them use a paperclip to measure the object. Then have them measure objects (chalkboard, bookcase, door width) using the length of their arm. Answers will be different. Discuss the need for standard measure. Then have them use metric measure to measure the objects. [3.5.4.1, 3.5.4.2, 3.6.1]	Jerry has gumballs in a plastic box. Right now there are 58 gumballs in the box shown below 
3.6.2 Make estimates to compare to actual results of computations.	1. Estimate the outcomes of problems/ experiments, complete the task, and compare the results with the prediction.	Materials: index cards, paper bag, pencils, paper, calculator. Write one of the following numbers on each of 10 index cards: 27,112,38,211,63, 85,77,45,149,156. Put the cards in the bag and mix them up. One person draws out 2 cards, estimates the sum, and records the estimate on a piece of paper. The other person uses the calculator to find the sum, and records it by the estimate. Compare the two sums and decide if the calculator answer is reasonable. [3.1.3.2, 3.6.2]	
3.6.3 Recognize situations where only an estimate is required.	1. Explore the meaning of large numbers through such activities as estimating the grains of rice in a coffee can, the number of letters on a page, the length of string in a millimeter… 2. Explore quantitative information which will help to relate personal experiences to the meaning of million.	Suggest that the class could make a cube 100 squares long, 100 squares wide and 100 squares high. Discuss how to do this and decide on one approach. As they discuss their ideas, ask, "If we made such a cube, how many small cubes would be in it? Why?" [3.4.3, 3.5.3.2, 3.5.4.3, 3.6.3]	

<table>
<tr>
<td>3.5.6. Apply proportions to scale drawings and direct variation</td>
<td>1. Ratio
2. Proportion
3. Percent
4. Similar figures
5. Similar polygons: ratio of perimeters and areas
6. Direct variation</td>
<td>In 200 BC, Eratosthenes devised this ingenious method for measuring the distance around the earth. To measure the circumference of the Earth Erotosthenes used his knowledge of geometry, particularly the theorem, Parallel lines cut by a transversal form congruent alternate angles. He determined that at noon during the summer solstice in the city of Syene (Egypt) a vertical rod did not cast a shadow, while in Alexandria (500 miles away) the vertical rod cast a shadow that formed a 7°12" angle. With this information, he was able to calculate the circumference of the earth to within 2% of its actual value.



In the drawing above light rays travel parallel to each other, ∠CAB and ∠B in the diagram are congruent alternate interior angles. Thus the distance between Syene and Alexandria is proportional to the distance around the earth as the ratio of the number of degrees in ∠B is to the number of degrees in a circle. What was Eratosthenes' estimate of the circumference of the earth?
[3.5.9.1; 3.5.6.1; 3.5.6.2; 3.5.6.5; 3.5.6.6]</td>
</tr>
</table>

3.6.8 Determine probabilities of simple events.	1. Determine the number of ways an event can occur. 2. Use fractional notation to express the probability of an occurrence 3. Explore problems that involve a systematic identification of ordered arrangements using models, pictures, lists, or tree diagrams.	The spinner below was used by Jodie's class for the school fair: A. If the spinner is spun once, what is the probability of the spinner landing on an even number? Answer as a fraction. B. If the spinner is spun a second time, what is the probability of the spinner landing on a number that is divisible by 3? [3.6.8]	

3.7 Patterns/Functions 3-4

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS	CONCEPTS/SKILLS	SAMPLE TASKS	ASSESSMENT ITEMS
3.7.1 Recognize, describe, extend, and create a wide variety of patterns.	1. Investigate the patterns in the 10 x 10 multiplication table. 2. See 3.1.2 and 3.4.1	Draw a 3–rectangle on the board. (Refer to the drawing as a 3-rectangle because it is made from 3 small rectangles.)	* * * * * * * *

		the circumference of the wheel. To do this right the odometer has to be "set" for the right wheel circumference. If it is set for the wrong circumference, it's readings are consistently too high or too low. Before Paul's experiment he estimated that his wheel circumference was 210 cm. Then he set his odometer for this circumference. Use the results of his experiment to find a more accurate estimate for the circumference. [3.5.8.1]
3.5.9. Use geometric relationships in relevant measurement problems involving geometric concepts.	1. Similar polygons: ratio of perimeters and areas 2. Similar figures 3. Comparison of volumes of similar solids	Phil works for a printing company. He has been given the job of ordering boxes to ship dictionaries. The books are 3 inches thick, 6 inches wide, 10 inches long and weigh 4 pounds. He wants to have boxes made which will hold 2 dozen books, with no wasted space in the box. No dimension of the boxes can be greater than 36 inches. What should be the dimensions of the boxes he orders? Can it be done? If not explain why. [3.5.9.3]

3.6 Uncertainty Math A

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

ASSESSMENT ITEM [3.6.3; 3.6.4.]

(1 point)

Erica cannot remember the correct order of the four digits in her ID number. She does remember that the ID number contains the digits 1,2, 5, and 9. What is the probability that the first three digits of Erica's ID number will all be odd numbers?

(1) ¼ (2) 1/3 (3) ½ (4) 3/4

		the circumference of the wheel. To do this right the odometer has to be "set" for the right wheel circumference. If it is set for the wrong circumference, it's readings are consistently too high or too low. Before Paul's experiment he estimated that his wheel circumference was 210 cm. Then he set his odometer for this circumference. Use the results of his experiment to find a more accurate estimate for the circumference. [3.5.8.1]
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3.7.4 Solve for an unknown using manipulative material.	1. See 3.1.1, 3.1.2, 3.3.2 and 3.3.4	2. triangles to build a square congruent to the large one. Ask students to share different arrangements. Point out that each of the arrangements represents a combination of the same pieces. Now have students work in groups to: "Find all the different combinations of pieces that can be used to build a square congruent to the large square, and record each." Have each group describe and draw one combination until all possibilities are exhausted. Display the solutions so visual comparisons can be made. [3.1.1, 3.1.2, 3.3.2 and 3.3.4]	are in the figure you draw? Write one or two sentences to describe how the figure is changing.
3.7.5 Use a variety of manipulative materials and technologies to explore patterns.	1. See 3.1.2	Materials: calculator Searching for the pattern. Your calculator can help you in your search for the next two numbers in each sequence. Sometimes only one operation will do an sometimes you will need to use more than one. A. 18 9 4.5 2.25 1.125 ___ ___ B. 19 20.5 22 23.5 25 ___ ___ C. 132 99 74.25 55.6875 41.765625 ___ ___ [3.1.2, 3.1.3, 3.2.2, 3.3.1]	

		keeps. Rental Plan 5 Each month, you get to roll two dice. If your total is 4 or less, you pay $1000. Otherwise, you pay nothing. Rental Plan 6 Each month, you pick a card, at random, from a Standard deck (no jokers). If it is an ace, you pay $2000. If it is a numbered card (2 through 10), your rent is the number you picked multiplied by 50. If you pick a face card, You pay nothing. [3.6.2.1; 3.6.3.1]
3.6.3. Use the concept of random variable in computing probabilities	1. Mutually exclusive events 2. Counting Principle 3. Sample space 4. Probability distribution 5. Probability of the complement of an event	Two dice are tossed and the sum of the numbers that come up are recorded. What are all the possible sums and what is the probability of each sum. Determine if the requirements of a probability distribution are met in this example. [3.6.3.3; 3.6.3.4; 3.4.1.10]
3.6.4. Determine probabilities using permutations and combinations	1. Counting principle 2. Factorial notation 3. Permutations: ${}_nP_n$ and ${}_nP_r$ 4. Combinations: ${}_nC_n$ and ${}_nC_r$	A home security device has ten buttons. When three different buttons are pushed in the proper sequence the alarm does not go off. No button can be pushed twice. If you forget the correct code, what is the probability that by randomly pushing three of the buttons you will pick the correct code. [3.6.4.3; 3.6.2.2]

		other instruments.	

	addition-subtraction method	
3.7.2. Apply linear and quadratic functions in the solution of problems	1. Graphic and algebraic solutions of linear and quadratic functions in the solution of problems	The trajectory of a baseball after it leaves the bat can be described by the equation $h(x) = -0.05x^2 + 5.4x$ where $h(x)$ denotes the height of the ball when it has traveled x yards from home plate. Have students graph the function and determine the greatest height reached by the baseball, the horizontal distance of the ball from home plate when it reaches its greatest height and the horizontal distance traveled by the ball. [3.7.1.4; 3.7.2.1; 3.7.4.1]
3.7.3. Translate among the verbal descriptions, tables, equations and graphic forms of functions	1. Translate linear and quadratic functions, systems of equations, inequalities and quadratic linear pairs between representations that are verbal descriptions, tables, equations, or graphs	"Grab bag" assortments of fishing lures all contain $6 worth of lures. Selection A contains three plugs and one jig. Selection B has a plug, two spoons, and a jig. Selection C has five jigs and two spoons. What is the cost of each type of lure? Solve this problem both graphically and algebraically and explain how the solutions can be found on the graph. [3.7.3.1; 3.7.1.4; 3.7.1.5; 3.7.2.1; 3.7.4.1]
3.7.4. Model real-world situations with the appropriate function.	1. Determine and model real-life situations with appropriate functions	In the fact book *The Hidden Game of Baseball*, John Thorn and Pete Palmer present the following formula for determining the probability how many runs a particular player will make. Runs = .46(singles) + .8(doubles) + 1.02(triples) + 1.4 (home runs) + .33(walks + hit-by-pitches) + .3(stolen bases) - .6(caught stealing) -.25(at bats-hits) -.5(outs on base) Have students discuss what this equation means and whether it is reasonable. Why or why not? [3.7.4.1.; 3.7.3; 3.7.1]

MATHEMATICS CORE CURRICULUM GUIDE FOR GRADES 5-6

Standard 3: Mathematics

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability and trigonometry.

INTRODUCTION

Problem solving should be integrated throughout the standards. The development of problem solving skills should be a major goal of the mathematics program at every grade level. Students should engage in many problem-solving situations and have the opportunity to reflect upon their solutions. They should be actively involved individually and in groups in exploring, conjecturing, analyzing, and applying mathematics in both a mathematical and a real-world context. They should have opportunities to express their understanding in a variety of modes (diagrams, graphs, words, symbols, numbers and manipulatives). They should engage in hands-on conceptual learning and use appropriate technology for computation and exploration. However, the use of technology should not be a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

The curricular focus for grades 5 and 6 continues emphasis on the study of whole numbers, decimals, fractions and integers. Students should reason in spatial contexts, with proportions, from graphs, inductively and deductively. They should develop number and operation sense, create algorithms and procedures, use estimation both in solving problems and in checking the reasonableness of results. They should be able to identify and use functional relationships. Develop and use tables, graphs and rules to describe situations. They should develop an understanding of variables, expressions and equations and begin to investigate inequalities and nonlinear equations. Students use statistics to describe, analyze, evaluate and make decisions. They should be able to create experimental models of situations involving probabilities and start investigating theoretical models. They should develop an understanding of geometric objects and relationships and use geometry, measurement and estimation to solve problems.

Manipulatives are a crucial element of instruction at this level especially when new concepts are being introduced. It is not necessary to devote large amounts of money on the purchase of commercial manpualtives. There are a variety of objects like rulers, meter sticks, tape measures, protractors, and graph paper that are inexpensive and easily obtainable. Spinners can be made with a paper clip and a pencil point. Compasses can be made with string and a pencil. Fraction tiles, square counters, pattern tiles, tangrams, and pentominoes can be made from construction paper. Base 10 place value models no longer need be proportional but can include more abstract models as with chip trading activities. Versatile commercial materials include cubes, geoboards, dice, capacity measuring devices, and mirrors. Calculators should be able to perform fraction operations, powers and give very large or very small decimal numbers in scientific notation.

graphing and can be useful for developing conceptual understanding for topics such as successive approximations, graphical representations for solving equations and inequalities, trigonometric, algebraic, exponential and logarithmic graphs and problem solutions, statistics and probability problems. In addition, computers can be useful for exploring 2-D and 3-D figures.

Assessment of learning should be an integral part of instruction to inform the teacher of the effectiveness of instruction as well as student progress. Students should be expected to express their understanding with graphs, symbols and with written and oral descriptions. Class discussions should not be confined to recall questions but should evolve around open-ended questions that get students to explain their ideas.

The performance indicators, concept/skills, sample tasks and assessment items that follow are suggestions and meant to be a guide for use by local education agencies in developing their own curriculum. The order and placement of topics does not imply an expectation of how classroom instruction should be sequenced. They are not meant to be restrictive. We have used the following sources for ideas for sample tasks and assessment items. Teachers may wish to refer to them form more ideas.

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3.1 Mathematical Reasoning 5-6
Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

ASSESSMENT ITEM [3.1.1; 3.3.5; 3.5.1]
(3 point item)
It's Sunday and you're going to meet your friends for lunch and a movie. You have to leave your home at 11:30 AM. Your parents say you can't go until you finish your work. Your work includes your homework and your Saturday chores:

40 minutes of math homework
30 minutes to clean your room
15 minutes to fold the laundry
5 minutes to take out the garbage
60 minutes to eat and get ready to go

A) At what time should you get started doing your work?
Show all the math you did to figure this out.

Answer______________AM

B) Describe how you would use your time between when you wake up and when you leave at 11:30 AM to go to lunch and the movie.

Performance Indicator	Concepts/ Skills	Sample Tasks
3.1.1. Apply a variety of reasoning strategies.	1. Apply basic computational skills to problems from other subject areas. 2. Solve problems that illustrate the use of fractions (also decimals) to problems. 3. Write and solve open sentences while ?	Have students solve the following problem in groups and report to the class the strategy they used: A census taker asked the farmer the ages of his three daughters. The farmer told him that the product of their ages is 72 and the sum of their ages is the house number. The census taker performed some ?

?, Not readable

		one step. If there are not enough steps to the proof for each student in the group, extra "clues" can be given as "wild cards".
3.1.5 Construct indirect proofs.	1. Euclidean, analytic, logical and trigonometric indirect proofs.	Prove that $\sqrt{2}$ is an irrational number. [3.1.5.1; 3.2.1.; 3.2.3]

Assessment Item [3.1.4]

Prove that if two secants are drawn to a circle from an external point, the product of the lengths of one secant and its external segment is equal to the product of the lengths of the other secant and its external segment.

3.2 Number and Numeration

Students use number sense and numeration to develop an understanding of multiple uses of numbers in real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

Performance Indicators	Concepts/Skills	Sample Tasks
3.2.1 Understand and use rational and irrational numbers.	1. Determine from the discriminate of a quadratic equation whether the roots are rational or irrational. 2. Rationalize denominators. 3. Simplifying of algebraic fractions with polynomial denominators. 4. Factor out common monomials and factor the resulting polynomial by factoring polynomials and the difference in two squares. 5. Simplify complex fractions.	Physicists tell us that the altitude h in feet of a projectile t seconds after firing is $h = -16t^2 + v_u t + h_0$ where v_u is the <u>upward</u> component of its initial velocity in feet per second and h_0 is the altitude in feet from which it is fired. A rocket is launched from a hilltop 2400 feet above the desert with an initial upward velocity of 400 feet per second. When will it land on the desert? Discuss what the discriminate can tell you about the solution to this problem. Then use the quadratic equation to find the solution and explain you answer. [3.2.1.1; 3.2.4.3]
3.2.2 Recognize the order of the	1. Give rational approximations of	Explain the difference between the numbers in each

		area is 18 square inches). Students can count around the rectangles to find the perimeters. Have students generalize ways to calculate the area and perimeter of rectangles and squares in the form of a formula. (3.1.3.1; 3.5.3.2)
3.1.4. Justify conclusions involving simple and compound (i.e., and/ or) statements	1. Study basic set ideas and terms, such as *universal set, subset, intersection, union, equivalent set and complement* in real situations.	Teacher draws two circles on the board that intersects. Label the overlapping loops with cards indicating values of different attributes. Let students take turns with examples and deciding in which region they belong. Pieces belonging in neither loop are placed outside. Let other students decide if the placement is correct and occasionally have someone else explain. Use terms like *universal set, intersection, union,* and *complement.* (3.1.4.1)

3.2 Number and Numeration 5-6

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

ASSESSMENT ITEM [3.1.2; 3.2.1; 3.4.7;]
(3 POINTS)
Design a spinner that has four regions with the following features:

Score value	*Fraction*
100 points	1/8
50 points	1/8
25 points	¼
10 points	½

Draw the spinner and label the sectors. Then explain your solution in words.

		Rise in Level = (Average change in water level per marble) x (Number of marbles). Students can also plot the water level changes for each trial as ordered pairs. In a proportional relationship they will get a straight line which passes through the origin (proportion is a linear relationship). Use the y axis for the rise in water level and the x axis for the number of marbles. (3.2.2.4; 3.3.7.1; 3.4.3.1; 1.1.1; 4.3.1 (physical setting)
3.2.3. Develop an understanding of number theory (primes, factors, and multiples).	1. Factoring techniques to determine common denominators	Using a calculator which operates on fractions enter two denominators as a fraction (Let the smaller number be the numerator). Students use the simplify key to get the fraction in lowest terms and keep track of what number was factored out each time. Have students explain how the calculator simplified the fraction and how the factors could be used to find a common denominator. (The simplify function factors out prime numbers starting with 2 and continues until all common factors are factored out. If students multiply all the factors by the product of the numerator and denominator of fraction in lowest terms, they will have the common denominator). (3.2.3.1; 3.3.1.3)
3.2.4. Recognize order relations for decimals, integers, and rational numbers.	1. Extend negative number notation to fractions. 2. Compare decimals and common fractions using the terms *greater than, less than, between* or *equivalent*. 3. Compare size of fractions using several methods.	Encourage students to work in pairs; one student draws a line segment of any length, selects and labels the "endpoints" with the smaller number on the left and indicates the placement of missing numbers. The other student fills in the missing number and explains how they knew what number was indicated. This activity can focus on whole numbers,

3.3 Operations

Students use mathematical operations and relationships among then to understand mathematics.

Performance Indicators	Concepts/Skills	Sample Tasks
3.3.1 Use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions.	1. Laws of exponents. 2. Complete operations on fractions with polynomial denominators. 3. Add and subtract rational fractions with monomial and binomial denominators.	A googol is a 1 followed by 100 zeros, and a googolplex is a 1 followed by a googol of zeros. Express these two numbers as powers of 10. [3.3.3.1]
3.3.2 Develop an understanding of and use the composition of functions and transformations.	1. Understand the general concept and symbolism of the composition of transformations. 2. Apply the composition of transformations (line reflections, rotations, translations, glide reflections). 3. Identify graphs that are symmetric with respect to the axes or origin. 4. Isometries (direct, opposite). 5. Applications to graphing (inverse functions, symmetry). 6. Define and compute compositions of functions. 7. Functions (inverse, exponential, logarithmic)	The Environmental Protection Agency has determined that in a certain section of the country the average level of air pollution is $0.5\sqrt{P} + 10{,}000$ parts per million (ppm), where P is the population. The 1980 census predicts that the population *t* years after 1980 will be 7ooo + $40t^2$. A - Express the pollution level *t* years after 1980 as a composite function and reduce the composite function to a function of *t*. B - What pollution level can be expected in 1990? 2000? [3.3.2.6]
3.3.3 Use transformations on figures and functions in the coordinate plane.	1. Apply transformations (line reflection, point reflection, rotation, translation and dilation) on figures and functions in the coordinate plane. 2. Use slope and midpoint to demonstrate transformations. 3. Use the ideas of transformations to	On graph paper, set up a coordinate system for each figure and graph the figure by plotting coordinates and connecting adjacent vertices. Sketch the reflection of each shape over the line y=x. a. (3,2), (-1,-4), (7,2), (-2,3) b. (1,7), (4,5), (6,-1)

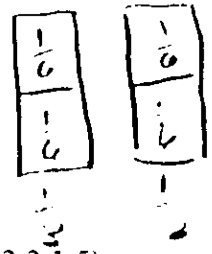
3.4 Modeling/Multiple Representation

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

Performance Indicators	Concepts/Skills	Sample Tasks
3.4.1 Represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs.	1. Express quadratic, circular, exponential and logarithmic functions in problem situations algebraically 2. Use symbolic form to represent an explicit rule for a sequence 3. Relate algebraic expressions to the graphs of functions 4. Definition and graph of an inverse variation (hyperbola).	Draw the graph y=48/x. Make a table using some integral values of x from x=-16 to x=16 ($x \neq 0$). Identify the graph. [3.4.1.4]
3.4.2 Manipulate symbolic representations to explore concepts at an abstract level.	1. Use positive, negative and zero exponents and be familiar with the laws used in working with expressions containing exponents. 2. In the development of the use of exponents, the students should review scientific notation and its use in expressing very large or very small numbers. 3. Rewrite the equality $\log_b a = c$ as $a = b^c$. 4. Solve equations using logrithmic expressions 5. Use the laws for performing calculations of exponents and logarithms	Prove: If x and n are real numbers, and $x > 0$, then $\log_a (x^n) = n\log_a x \quad a > 0, a \neq 1$ [3.4.2.5]

		1/8 fraction tiles and note that the two parts of ¾ are each once again 3/8.  (3.3.1.5; 3.3.4; 3.3.5.1)
3.3.2 Explore and use the operations dealing with roots and powers	1. Use exponential notation to reinforce place value concepts 2. Use the exponential form of powers of 2, 3,5, and 10 and relate these forms to factoring. 3. Explain briefly the use of exponents in scientific notation, using only positive exponents.	Have students work in groups to solve the following problem: Leon asked his father to give him an allowance if he performed odd jobs around the house. The pay scale Leon suggested to his father was that the first week his allowance would be only one cent. The second week it would be two cents, 4 cents the third week and so on. Each successive week his allowance would be twice the amount of the previous week. His father thought this arrangement was very much to his advantage and was pleased to accept. Leon was also very happy with this deal. Find out how much allowance Leon would get in the fourth week, the eight week and the 20^{th} week. How could you figure out what Leon's allowance would be for any given week? Was this a good deal for Leon or for his father? Why? (3.3.2.2; 3.1.2.1; 3.7.1.1; 3.7.2.2;)
3.3.3. Use grouping symbols (parentheses) to clarify the intended order of operations.	1. Use the conventional rule for order of operations.	Have students express the numbers one through ten by combining 4's with any mathematical operation. They must make sure they use the order of operations correctly and use parenthesis when need to show exceptions for example- 5 = (4 x 4 + 4)/ 4 Encourage students to find a variety of solutions. (3.3.3.1)

complex numbers.	conjugates.	real number. [3.4.7.1]
3.4.8 Model and solve problems that involve absolute value and vectors.	1. Solve using the polar form of complex numbers. 2. See 3.4.6.1.	The amount of alternating current in a circuit is restricted by the impedance Z (in ohms). If a resistor, inductor, and capacitor are connected in series the impedance is $Z = R + j(X_L - X_C)$, with magnitude $\|Z\|$ and direction Θ given, respectively, by $\|Z\| = \sqrt{R^2 + (X_L - X_C)^2}$ and $\theta = \tan^{-1}((X_L - X_C)/R)$ Provide students with various values for R, X_L *and* X_C and have them compute the magnitude and direction of Z (to the nearest whole number) for each set of data. [3.4.9.1; 3.4.6.1]
3.4.9 Model quadratic inequalities both algebraically and graphically.	1. Model algebraically and graphically inequalities such as $x^2 - 5x - 6 < 0$ to find the possible solutions.	See example in 3.4.5
3.4.10 Model the composition of transformations.	1. The composition of two line reflections when the two lines are parallel 2. The composition of two rotations about the same center. 3. The composition of two translations. 4. The composition of a line reflection and a translation in a direction parallel to the line of refection (glide refection)	Give students cut out triangles. Have them draw a line and put a point on it for a vertex (straight angle) by doing translations with the triangle, students are to show that the sum of the measures of the angles of a triangle is 180°. Have students list their translations in order (The translation for the first two angles can be done with a slide. The third angle can be done with a composition of line rotation and slide. Have students prove that the translations are legitimate using rules of transformations and parallel lines. [3.4.10.1; 3.4.10.3]
3.4.11 Determine the effects of changing parameters of the graphs	1. Be able to sketch the effects of changing the value of a in the function	The graph of a function can be transformed in a number of ways. We will consider three: Vertical

		produce this result.  (3.3.5.1; 3.3.1.5)
3.3.6. Develop appropriate proficiency with facts and algorithms.	1. **Ensure quick recall and mastery of** basic addition, subtraction, multiplication and division facts. 2. Develop strategies for mental math	One student is allowed to use a calculator and the other calculates mentally. The person who gets the correct answer first gets 1 point. The player with the higher score at the end wins. Have students discuss each item and why the person who got the point may have an advantage. 1. 2 + 9 + 16 + 18 + 14 _+ 1 + 10 2. 14 + 9 + 17 + 23 + 16 + 21 + 40 3. 31 + 18 + 10 + 19 + 34 + 2 + 16 4. 91 + 92 + 100 + 97 + 98 + 93 + 99 5. 3 + 8 + 9 + 10 + 11 + 12 + 17 6. 4 + 15 + 11 + 20 + 16 + 9 + 5 7. 43 + 24 + 8 + 17 + 32 + 26 + 10 8. 75 + 83 + 25 + 96 + 17 + 4 + 50 (3.3.6.1; 3.3.5; 3.3.4)
3.3.7. Apply concepts of ratio and proportion to solve problems.	1. Use ratio and proportion concepts to solve problems.	Give each group of students seventy or eighty counters. Show them three triangular numbers and that the terms and trinumbers are as below Term | Trinumber 1 | 3 2 | 6 3 | 9 4 | ? 5 | ? Challenge the students to create a written description of

trigonometric, and exponential functions to model real-world relationships.	relationship can be represented by a polynomial, rational, trigometric or exponential function. 2. Solve real-world problems by using polynomial, rational, trigometric and exponential functions.	film canisters (For directions see <u>Rockets</u>: A Teacher's guide with Activities in Science, Mathematics and Technology by NASA) and using water and baking soda for fuel.Have students make an astrolabe to measure the angle of altitude of the rockets assent. If students are a known distance from the rocket when they determine the angle of altitude, they can use TanΘ = Opposite/adjacent to determine the height the rocket reached by adding that result to the distance of their own eye level from the ground. [3.4.12.2}
3.4.13 Use algebraic relationships to analyze the conic sections.	1. Write the equation of a circle with a given center and radius and determine the radius and center of a circle whose equation is in the form $(x - h)^2 + (y - k)^2 = r^2$. 2. Recognize an equation in the form $y = ax^2 + bx + c$, $a \neq 0$ as an equation of a parabola and ◆ be able to form a table of values in order to sketch its graph ◆ find the axis of symmetry ◆ determine the abscissa of the vertex to provide a point of reference for choosing the x-coordinates to be plotted ◆ find the y-intercept of the parabola.	Write an equation of a circle with a center T(4,-3) And radius 3, using the distance formula. [3.4.13.1}
3.4.14 Use circular functions to study and model periodic real-world phenomena.	1. Use the concept of the unit circles to solve real-world problems involving ◆ Radian measure	After students have graphed circular functions, have music students in the class play their instruments one at a time and show the sounds on

3.4.2. Use maps and scale drawings to represent real objects or places.	1. Make scale drawings like floor plans using centimeter grids to relate scale to ratio.	Have students work in teams to make a scale drawing of the classroom including the desks and tables each using a different scale on centimeter grid paper. Show all the drawings and have students vote on which scale they like the best and explain why. (3.4.2.1)
3.4.3. Use coordinate plane to explore geometric ideas.	1. Explore measurement and vocabulary of geometric figures using a concrete discovery approach using geoboards, lattices and graphing paper. 2. Continue graphing with ordered pairs of numbers.	A linear unit on a geoboard is designed as-  A square unit is designated as-  Students are given a fixed perimeter (e.g. 8) and are asked to find the dimensions and area of as many rectangles as possible. Which rectangle has the greatest area? After students try several other examples ask them to generalize how to find the rectangle with the greatest area given a specific perimeter. [3.4.3.1; 3.1.3.1]
3.4.4. Represent numerical relationships in one- and two-dimensional graphs.	1. Graphs: circle, bar, line, pictograph, and stem and leaf. 2. Compare histogram, line, picture, circle graphs, and stem and leaf which present the same information, and note the advantages and disadvantages of each. 3. Correlate graphing with computational skills.	Given a set of 1-variable data have students create a histogram (for example the ages of the presidents when they had their first inauguration.). Ask students if they can find the mode, mean and median from the graph. Show the students how to display the same information in a stem and leaf plot. Put the histogram over the stem and leaf to show they create the same graph. Ask students once again how they would find the mode, median and mean on the graph. [3.4.4.1; 3.4.4.2]
3.4.5. Use variables to represent relationships	1. Write and solve open sentences dealing with inverse operations, using letters as well as frames as placeholders.	Remind students that the formula for the circumference of a circle is $C = \pi D$. Have them use string and a ruler to find the circumferences of a number of bottle tops and canister tops. Have them use the formula to find the radius of each of the circles so they can find the area of

		advance of 12 feet. What angle does the roofline make with the horizontal in this case? [3.5.2.1]
3.5.3 Derive and apply formulas relating angle measure and arc degree measure in a circle.	1. Express angle measure in terms of degrees and radians. 2. Reference and coterminal angles 3. Understand the derivation and apply formulas for sine, cosine, tangent and their reciprocal trigonometric function. 4. Sum and difference of two angles 5. Double and half angles. 6. Vectors	Give students a cone-shaped drinking cup. Have the students cut the side from the brim to the apex of the cone and flatten out the cup. The shape of the flattened surface will be a circle with a sector missing. Ask them to use the shape and the ideas of unit circles to help them find the surface area of the cone. [3.5.3.1; 3.5.3.2; 3.5.3.3; 3.5.3.4]
3.5.4 Prove and apply theorems related to lengths of segments in a circle.	1. Prove and apply theorems related to arcs, chords, tangents, secants and angles. 2. Prove theorems related to congruence.	Prove that any trapezoid inscribed in a circle is an isosceles trapezoid; that is, at least one pair of opposite sides are equal. [3.5.4.1; 3.5.4.2; 3.1.4.1]
3.5.5 Define the trigonometric functions in terms of the unit circle.	1. Sine, cosine, tangent and their reciprocal functions on the unit circle. 2. Radian measure. 3. Coordinates of a point on the unit circle expressed as $\cos\theta$ $\sin\theta$. 4. Special angles of 30°, 45° and 60°. 5. Reference angles. 6. Amplitude and period. 7. Reflections in the line $y = x$. 8. Inverse functions.	Sketch the six basic trigonometric functions and their inverses on the graphing calculator. Superimpose the functions with their inverses on a graphing calculator. [3.5.5.1; 3.5.5.6; 3.5.5.8]
3.5.6 Relate trigonometric relationships to the area of a triangle and to general solutions of triangles.	1. Application of the sine function in the solution of the area of a triangle. 2. Law of sines ◆ Finding a side given ASA or Aas	Prove that if ABC is a right triangle, the Law of Cosines reduces to the Pythagorean theorem. [3.5.6.3;3.1.4.1]

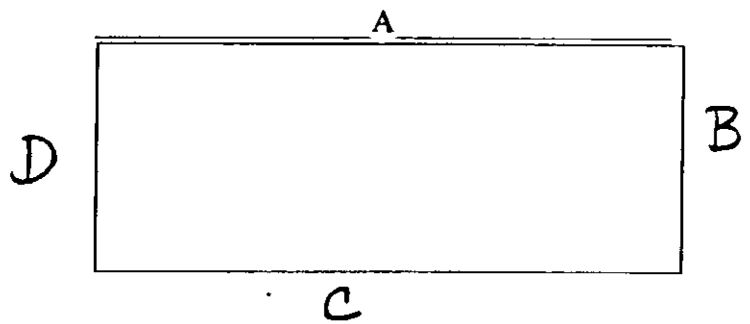
for basic geometric constructions.	ruler, do various constructions activities.	protractor to make a hexagon. They will need to know that there are 360 degrees in a circle. [3.4.10.1; 3.4.9.1, 3.5.2.2]

3.5. Measurement 5-6

Students use measurements in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

ASSESSMENT ITEM [3.1.2; 3.3.1; 3.4.2; 3.5.1; 3.5.3; 3.7.6]
(3 POINTS)
Central Park City Playground is getting a new fence and grass. The scale drawing below shows the shape of this playground.

A) Use your centimeter ruler to find the length of each side. If each centimeter of the scale drawing represents 3 meters, find the actual length of each side of the playground and the total amount of fencing needed to go around the playground.



Side	Measurement	Actual Length
A	cm	m
B	cm	m
C	cm	m
D	cm	m

B) Find the total meters of fencing needed to go around the playground. Explain your answer.

Answer________________

C) If the entire playground will be planted in grass, how many square meters will need to be planted? Explain your answer.

	equations and inequalities 3. Check solutions of permutation problems using combination formulas as a check and vice versa 4. Using proof as a check on the validity of geometric constructions 5. Compare histograms with formula derived solutions for mean, median, variation and standard deviation 6. Use the Quotient Identities, Reciprocal Identities, and the Pythagorean Identities to establish that other equations are identities	
3.6.2 Judge the reasonableness of a graph produced by a calculator or computer.	1. Determine the effects of changing the parameters of graphs of linear, quadratic, trigonometric, exponential, and circular functions 2. See 3.4.11.1 3. See 3.4.11.2	A rich philanthropist, who loved mathematics, agreed to sponsor an 18 hole golf tournament at the local country club. In order to enter, a contestant had to pay 2 cents and select either a linear, quadratic, or exponential formula to calculate how many CENTS he/she would receive for a winning hole. In each of the following formulas, X represents the number of the winning hole. linear, $Y=2X$; quadratic, $Y=X^2$, exponential, $Y=2^X$. Why bother entering if the payoff is in pennies? Use your graphing calculator to investigate. Describe numerically how the amounts change from one hole to the next for each method. Which method would you select on your entry from and why?

<table>
<tr><td></td><td></td><td><table><tr><th>Side</th><th>Perimeter</th><th>Area</th><th>Ratio P</th><th>Ratio A</th></tr><tr><td>1</td><td>4</td><td>1</td><td>1:4</td><td>1:1</td></tr><tr><td>2</td><td>8</td><td>4</td><td>1:4</td><td>1:2</td></tr><tr><td>3</td><td>12</td><td>9</td><td>1:4</td><td>1:3</td></tr></table>What patterns do you notice? How might you be able to find a perimeter of any given side? How might you find an area of any given side?
[3.1.3.1; 3.2.2.4; 3.2.2.5; 3.4.4.1; 3.4.5; 3.5.3.2]</td></tr>
<tr><td>3.5.4. Use statistical methods and measures of central tendencies to display, describe and compare data.</td><td>1. Consider uses and misuses of averages in interpreting data.</td><td>Provide students with a set of salaries for a company that includes both management and other workers. Present the problem that the workers union is asking for a raise and justifying it on an average wage (which might be the median or mode). Management claims that there should not be a wage increase because they have calculated a different average wage which is quite a bit higher (probably the mean). Have the students determine if one of them is not telling the truth and propose and justify a fair solution.
[3.5.4.1; 3.1.2.1]</td></tr>
<tr><td>3.5.5. Explore and produce graphic representations of data using calculators/ computers</td><td>1. Compare graphs that can be produced on a graphing calculator: histogram, line, box and whisker.</td><td>Using a graphing calculator which can produce histograms and box and whisker plots, provide students with data like the homeruns hit by Babe Ruth and Lou Gehrig and display the data on a bar graph and a box and whisker plot. Ask which provides a better comparison of the homeruns hit by the two Yankees. (Ruth's median will be at Gehrig's lower quartile. Ruth's lower and upper limits are higher than Gehrig's. The range and interquartile range of Ruth's data are not as wide as Gehrig's indicating less dispersion and more consistency.
[3.5.5.1; 3.5.4; 3.4.4.2; 3.1.2.2]</td></tr>
<tr><td>3.5.6. Develop critical judgment for the reasonableness of</td><td>1. Relate metric units to customary units via approximations.
2. Make real world comparisons of</td><td>Pose the following problem and ask groups of students to explore ways to answer each question.
◆ How much water do you drink in one year? (Enough</td></tr>
</table>

		at 50% green? If you think the principal is justified in being suspicious what is the smallest occurrences of 75% that would be required to convince you? [3.6.6.2; 3.6.3; 3.6.4; 3.6.5]
3.6.7 Use a Bernoulli experiment to determine probabilities for experiments with exactly two outcomes.	1. Definition of a Bernoulli experiment 2. Case where r successes are assumed to occur first 3. General case	If the problem can be regarded as a Bernolli experiment, state the values of n, p, q, and r, and give the answer in symbolic form. If the problem cannot be regarded as a Bernoulli experiment, explain why. ♦ Four balls are drawn with replacement from an urn containing 4 red balls and 2 white balls. What is the probability of drawing exactly 2 red balls? ♦ Four balls are drawn without replacement from an urn containing 4 red balls and 2 white balls. What is the probability of drawing exactly 2 red balls? [3.6.6.1; 3.6.7.1]
3.6.8 Apply the concept of random variable to generate and interpret probability distributions.	1. Normal Distribution 2. Binomial Theorem (Pascal's Triangle)	The cause of death is related to heart disease in 52% of the cases studied. Find the probability that in 500 randomly selected cases, the number of heart-disease-related deaths differs from the mean by more than two standard deviations. [3.6.8; 3.5.7]
3.6.9 Create and interpret applications of discrete and continuous probability	1. Measures of central tendency 2. Use of Σ-notation	Which would you expect to have a higher variance: The IQ scores of a class if 25

3.6.2. Use estimation to solve problems for which exact answers are inappropriate.	1. Develop an awareness of when an estimation is more appropriate than an exact answer.	Have students discuss in groups the following questions. *Is an estimate enough when:* The waitress figures sales tax? The waitress finds the total bill? The customer figures a 15% tip? The customer checks the bill? [3.6.2.1; 3.1.2.1; 3.2.2.1]
3.6.3. Estimate the probability of events.	1. Make predictions based on sample data 2. Arrangements and Combinations.	Have students us the Tree Diagram technique for describing various combinations of outfits (choose from two pairs of slacks, four ties, six shirts), menus (choose from two beverages, three sandwiches, two desserts), team players, committee members, etc.) to estimate the probability of any one combination being picked at random. (3.6.3.1; 3.6.3.2; 3.4.6.1)
3.6.4. Use simulation techniques to estimate probabilities	1. Conduct simulations for experiments that cannot be determined theoretically or experimentally.	Have students simulate determining how many gum packages they would have to buy to get all four of four basketball stars that may be found in packages if there were an equal number of each star and one in each gum package. Students could create an appropriate spinner, randomly choose the names of four stars from a bag (with replacement)or use a page from the phonebook as a random number generator looking at the two digits only (00-24 = A; 25-49 = B; 50-74 = C; 75-99 = D) (3.6.5.1)
3.6.5. Determine probabilities of independent and mutually exclusive events.	1. Conduct and predict outcomes of experiments with independent events. 2. Introduce the symbolic representations of probability. $P(E) = f/n$	Students with a partner look at a number cube and determine the probability of rolling a 4 by noting how many fours there are on the cube compared to the total number of possible outcomes. $P(4) = 1/6$. Have the students roll the cube 20 times and tally the outcomes. Have them compare their prediction to their outcome. Teacher pools student results to examine whether a larger sample produces results closer to predicted results.

Performance Indicator	Concepts/Skills	Sample Tasks
3.7.1 Use function vocabulary and notation.	1. Definition of a relation 2. See 3.3.2.6 3. Determining if a relation is a function 4. Definition of inverse function 5. Notation for absolute value, compound functions 6. Expressing exponential functions as logs	See examp0le in 3.3.2
3.7.2 Represent and analyze functions using verbal descriptions, tables, equations, and graphs.	See 3.4.1.3;3.4.11; 3.4.13	See example in 3.4.1
3.7.3 Translate among the verbal descriptions, tables, equations, and graphic forms of functions.	See 3.4.3; 3.4.11; 3.4.13	See example in 3.4.13
3.7.4 Analyze the effect of parametric changes on the graphs of functions.	See 3.4.11; 3.4.13; 3.6.2.1	See example in 3.6.2.
3.7.5 Apply linear, exponential, and quadratic functions in the solution of problems.	See 3.4.12	See example in 3.4.12
3.7.6 Apply and interpret transformations to functions.	See 3.3.3	See example in 3.3.3
3.7.7 Model real-world situations with the appropriate function.	1. Characteristics of linear, quadratic, trigonometric, circular, exponential and logarithmic function.	Nita Pass is about to study for a mathematics exam. Nita knows that the test grade is a function of a number of hours studied and knows from past experience that 1 hour of studying will result in a grade of 60; 2 hours, in a grade of 74; and 7 hours in a grade of 84. 1-Show Nita that the grade is not a linear function of the number of hours studied. 2-Assume that the grade varies quadratically with the number of hours studied. Find the equation for the function, draw the graph (show important features: vertex and intercepts).

Performance Indicator	Concepts/ Skills	Sample Task
3.7.1 Recognize , describe, and generalize a wide variety of patterns and functions.	1. Continue to develop computation skills and the ability to recognize patterns.	Have students circle abundant numbers on a hundred board and describe the pattern on the hundred board. The sum of an abundant number's divisors is always greater than its double. For example, the sum of the divisors of 12 (1 + 2 + 3 + 4 + 6 + 12 = 28) is greater than 24 which is 12's double. (3.7.1.1; 3.2.3; 3.3.1;3.3.6.1)
3.7.2. Describe and represent patterns and functional relationships using tables, charts and graphs, algebraic expressions, rules, and verbal descriptions.	1. Correlate graphing with computational skills. 2. Use a variety of representations for the same functional relationship.	A tape measure is taped on a wall. One student releases a ball from specified distances measured from the bottom of the ball in centimeters (20 cm, 40 cm, 60 cm, 80 cm, and 100 cm.) A second student kneels to get a level view of the ball and tape measure and determines the bounce height at the bottom of the ball. The ratio of the drop height to the bounce height will be approximately the same for each kind of ball. The bounce height is proportional to the drop height. Students can be asked to represent the relationship with their data table, a verbal description, a rule, an algebraic expression and a graph. [3.2.2.4; 3.4.4.1; 3.7.2.23.7.4; 3.7.5.1]
3.7.3. Develop methods to solve basic linear and quadratic equations.	1. Find the missing value in a proportion in which three of the numbers are known and letters are used as placeholders. 2. Distinguish between linear and quadratic relationships.	Have students graph the perimeters of squares of different length sides and the areas of the same squares. They will notice that the graph of the perimeter ($y = 4x$ or $P = 4s$) is a straight line (proportional) and the graph of the area is a curved line ($y = x^2$ or $A = s^2$). [3.7.3.2; 3.2.2.4; 3.4.4.1; 3.4.5; 3.7.22; 3.7.4]

		g: $y=2x - 2$ f: $y=-1/2x + 2$ p: $y=x^2 + 1$ q: $y=(x+2)^2$ [3.7.14.1]
3.7.15 Apply the ideas of symmetries in sketching and analyzing graphs of functions.	1. Simplify the graphing of functions by using symmetries with respect to an axis, the origin, or some other point.	Find, if possible, a line of symmetry of the graph of each equation. If there is no line of symmetry, write none. $y=x^2 + 5$ $y = x^2 + 4x + 1$ $y = x$ [3.7.15.1]
3.7.16 Use the normal curve to answer questions about data.	1. Standard deviation for grouped data See 3.6.9	Find the variance and standard deviation for the data given in the following table. **Frequency table of time (in seconds) it takes the victim of a crime to call the police.** Time — Frequency 0-59 — 1 60-119 — 2 120-179 — 5 180-239 — 14 240-299 — 7 300-359 — 12 360-419 — 9 [3.7.16.1, 3.6.9.3, 3.5.7.3]
3.7.17 Develop methods to solve trigonometric equations and verify trigonometric functions.	1. Proving trigonometric identities 2. Solve first degree trigonometric equations 3. Solve quadratic trigonometric equations 4. Functions of sums and differences of angles 5. Double and half angle formulas	Let groups of students apply the following procedure to construct their own identities. Then they can prove each other's identities. 1. Start with a statement that is always true such as 1 = 1. 2. Replace part(s) of the beginning statement with an equivalent trigonometric expression such as $\sin^2\Theta$

3.7.6. Apply the concept of similarity in relevant situations.	1. Use concrete and artistic experiences to explain similarity and congruence in plane geometric figures	Students follow directions to fold and tear a piece of paper into tangram pieces testing for similarity and congruence of shapes and ways of determining whether shapes are similar or congruent. [3.7.6.1]
3.7.7. Use properties of polygons to classify them.	1. Classify polygons by properties and develop definitions.	Using the geoboard, students make a polygon of their choice with one rubberband. Teacher has a shape in mind (perhaps a parallelogram). Students decide if their shape is or is not an example of what the teacher has in mind and explains why. Students determine the properties of the shape the teacher has in mind from the examples and nonexamples. [3.7.7.1; 3.1.3; 3.4.3]
3.7.8. Explore relationships involving points, lines, angles, and planes.	1. Identify the intersection of planes and 3-dimensional figures.	Students make cubes, cones, prisms, cylinders and other shapes out of oil-based craft clay. Using a tool called "pianó wire" they can slice the clay models to investigate the resulting faces on the slices.
3.7.9. Develop and apply the Pythagorean principle in the solution of problems.	1. A right triangle contains one right angle. 2. The hypotenuse of a right triangle is opposite the right angle 3. The hypotenuse of a triangle is greater than the other two legs.	Give groups of students sets of straws cut to a variety of lengths and something with a right angle to use as a model for the right angle of their triangle. Have them make right triangles with their straws and record the lengths of the sides of each triangle. Ask each group to share any observations they have made about their triangles. [3.7.9.2; 3.7.9.3]